1. (3pts) Evaluate the integral $\displaystyle\int \frac{dx}{\cos x + \sin x + 1}$.

   Let $t = \tan\left(\frac{x}{2}\right)$. Then

$$\begin{aligned}\int \frac{dx}{\cos x + \sin x + 1} &= \int \frac{\frac{2\,dt}{1+t^2}}{\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} + \frac{1+t^2}{1+t^2}} \\ &= \int \frac{2\,dt}{2+2t} \\ &= \int \frac{dt}{1+t} \\ &= \ln|1+t| + C.\end{aligned}$$

2. (3pts) Evaluate the integral $\displaystyle\int \frac{dx}{x^4+x^2}$.

   For some A, B, C, D,

$$\frac{1}{x^2+x^4} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{1+x^2}.$$

   Multiplying both sides by x^2 and setting $x = 0$, we get that $B = 1$. Then

$$\frac{A}{x} + \frac{Cx+D}{1+x^2} = \frac{1}{x^2+x^4} - \frac{1+x^2}{x^2+x^4} = \frac{-x^2}{x^2+x^4} = \frac{-1}{1+x^2}.$$

   So $A = C = 0$ and $D = -1$. Therefore

$$\begin{aligned}\int \frac{dx}{x^2+x^4} &= \int \frac{dx}{x^2} - \int \frac{dx}{1+x^2} \\ &= -\frac{1}{x} - \arctan x + C.\end{aligned}$$

3. (4pts) Estimate the number of intervals required to approximate the value of the integral $\displaystyle\int_1^2 \frac{dx}{x}$ by Midpoint rule to within 0.0001.

   We have $f(x) = \frac{1}{x}$, so $|f''(x)| = \left|-\frac{2}{x^3}\right| = \frac{2}{x^3}$, which is bounded above by 2 on the interval $[1, 2]$. So take $K = 2$. Then we want that

$$|E_M| \leq \frac{K(b-a)^3}{24n^2} = \frac{1}{12n^2} \leq 0.0001.$$

   This is equivalent to $n^2 \geq \frac{10000}{12}$, or $n \geq \sqrt{\frac{2500}{3}}$.