Math 1B, Quiz 2 Section 211 GSI Carter

Solutions

1. (3pts) Evaluate $\int \sqrt{x^2+4}\,dx$.
   Let $x = 2\tan\theta$. Then $dx = 2\sec^2\theta\,d\theta$. So,
$$\int \sqrt{x^2+4}\,dx = 4\int \sec^3\theta\,d\theta.$$

2. (3pts) Evaluate $\int \frac{1}{w^2+3w-4}\,dw$.

$$\begin{aligned}\frac{1}{w^2+3w-4} &= \frac{A}{w+4}+\frac{B}{w-1}\\ 1 &= A(w-1)+B(w+4)\\ &= w(A+B)+4B-A\\ 5B &= 1\\ B &= \frac{1}{5}\\ A &= -\frac{1}{5}.\end{aligned}$$

   So,
$$\begin{aligned}\int \frac{1}{w^2+3w-4}\,dw &= \frac{1}{5}\int\frac{dw}{w-1}-\frac{1}{5}\int\frac{dw}{w+4}\\ &= \frac{1}{5}\ln|w-1|-\frac{1}{5}\ln|w+4|+C.\end{aligned}$$

3. (4pts) Evaluate $\int \frac{1}{\sqrt{x^2-6x+8}}\,dx$.
   Let $x-3 = \sec\theta$. Then $dx = \sec\theta\tan\theta\,d\theta$. So,
$$\begin{aligned}\int \frac{1}{\sqrt{x^2-6x+8}}\,dx &= \int \frac{\sec\theta\tan\theta}{\sqrt{\sec^2\theta-1}}\,d\theta\\ &= \int \sec\theta\,d\theta\\ &= \ln|\sec\theta+\tan\theta|+C\\ &= \ln\left|x-3+\sqrt{x^2-6x+8}\right|.\end{aligned}$$