Math 1B, Quiz 2 Section 203 GSI Carter

Solutions

1. (3pts) Evaluate $\int \sqrt{x^2-4}\,dx$.

   Let $x = 2\sec\theta$. Then $dx = 2\sec\theta\tan\theta\,d\theta$, and

$$\begin{aligned}
\int \sqrt{x^2-4}\,dx &= 2\int \sec\theta\tan\theta\sqrt{4\sec^2\theta-4}\,d\theta \\
&= 4\int \sec\theta\tan^2\theta\,d\theta \\
&= 4\int \sin^2\theta\cos^{-3}\theta\,d\theta \\
&= 4\int \frac{\sin^2\theta}{(1-\sin^2)^2}\cos\theta\,d\theta \\
&= 4\int \frac{u^2}{(1-u^2)^2}\,du \\
&= \frac{2u}{1-u^2} + \ln|1-u| - \ln|1+u| + C \\
&= \frac{2\sin\theta}{\cos^2\theta} + \ln|1-\sin\theta| - \ln|1+\sin\theta| + C \\
&= \frac{x\sqrt{1-\frac{4}{x^2}}}{2} + \ln\left|1-\sqrt{1-\frac{4}{x^2}}\right| - \ln\left|1+\sqrt{1-\frac{4}{x^2}}\right| + C.
\end{aligned}$$

2. (3pts) Evaluate $\int \frac{1}{w^2-4w+3}\,dw$.

$$\begin{aligned}
\frac{1}{(w-1)(w-3)} &= \frac{A}{w-1} + \frac{B}{w-3} \\
1 &= A(w-3) + B(w-1) \\
&= w(A+B) + (-3A-B) \\
-2A &= 1 \\
A &= -\frac{1}{2} \\
B &= \frac{1}{2}
\end{aligned}$$

   So,

$$\begin{aligned}
\int \frac{1}{w^2-4w+3}\,dw &= \frac{1}{2}\int \frac{dw}{w-3} - \frac{1}{2}\int \frac{dw}{w-1} \\
&= \frac{1}{2}\ln|w-3| - \frac{1}{2}\ln|w-1| + C.
\end{aligned}$$

3. (4pts) Evaluate $\int \frac{1}{\sqrt{x^2+4x+8}}\,dx$.

Let $x + 2 = 2\tan\theta$. Then $dx = 2\sec^2\theta\, d\theta$, and

$$\begin{aligned}\int \frac{1}{\sqrt{x^2+4x+8}}\, dx &= 2\int \frac{\sec^2\theta}{\sqrt{4\tan^2\theta+4}}\, d\theta \\ &= 2\int \frac{\sec^2\theta}{2\sec\theta}\, d\theta \\ &= \ln|\sec\theta+\tan\theta| + C \\ &= \ln\left|\frac{\sqrt{x^2+4x+8}+x+2}{2}\right| + C.\end{aligned}$$