1. (4pts) Solve using variation of parameters: $y'' - y = e^{2x}$.

   The characteristic equation for the homogeneous equation is $y^2 - 1 = 0$. So, let $y_1 = e^x$ and $y_2 = e^{-x}$. Then we have that

$$u_1' e^x + u_2' e^{-x} = 0$$
$$u_1' e^x - u_2' e^{-x} = e^{2x}.$$

   Adding these two equations, we obtain $2u_1' e^x = e^{2x}$, so that $u_1' = \frac{1}{2}e^x$. Therefore $u_1 = \frac{1}{2}e^x + c_1$. Since $u_2' = -u_1' e^{2x}$, $u_2' = -\frac{1}{2}e^{3x}$. Therefore $u_2 = -\frac{1}{6}e^{3x} + c_2$. So, the general solution to the differential equation is

$$y = u_1 y_1 + u_2 y_2 = c_1 e^x + c_2 e^{-x} + \frac{1}{2}e^{2x} - \frac{1}{6}e^{2x} = c_1 e^x + c_2 e^{-x} + \frac{1}{3}e^{2x}.$$

2. (6pts) Solve using any technique: $y'' + 2xy' + 2y = 0$.

   Let $y = \sum_{n=0}^{\infty} c_n x^n$. Then $y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$ and

$$y'' = \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} = \sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n.$$

   We also have that $2xy' = \sum_{n=0}^{\infty} 2nc_n x^n$. So,

$$0 = y'' + 2xy + 2y = \sum_{n=0}^{\infty}\left[(n+2)(n+1)c_{n+2} + 2nc_n + 2c_n\right]x^n.$$

   Then for each $n \geq 0$,

$$c_{n+2} = \frac{-2nc_n - 2c_n}{(n+2)(n+1)} = \frac{-2c_n}{n+2}.$$

   Then $c_{2k} = \frac{(-1)^k c_0}{k!}$ and

$$c_{2k+1} = \frac{(-2)^k c_1}{(2k+1)(2k-1)\cdots(3)(1)}.$$

   So, the general solution is

$$y = c_0 \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{k!} + c_1 \sum_{k=0}^{\infty} \frac{(-2)^k x^{2k+1}}{(2k+1)(2k-1)\cdots(3)(1)} = c_0 e^{-x^2} + c_1 \sum_{k=0}^{\infty} \frac{(-2)^k x^{2k+1}}{(2k+1)(2k-1)\cdots(3)(1)}.$$