Math 1B, Quiz 1 Section 211 GSI Carter

Solutions

1. (3pts) Evaluate $\int x^2 \ln x \, dx$.

   Letting $u = \ln x$ and $dv = x^2 \, dx$, $du = \frac{dx}{x}$ and $v = \frac{x^3}{3}$. So, $\int x^2 \ln x \, dx = \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} \, dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$.

2. (3pts) Evaluate $\int \theta^2 \cos\theta \, d\theta$.

   Letting $u = \theta^2$ and $dv = \cos\theta \, d\theta$, $du = 2\theta \, d\theta$ and $v = \sin\theta$. So, $\int \theta^2 \cos\theta \, d\theta = \theta^2 \sin\theta - 2\int \theta \sin\theta \, d\theta$.

   Letting $u = \theta$ and $dv = \sin\theta \, d\theta$, $du = d\theta$ and $v = -\cos\theta$. So, $\int \theta \sin\theta \, d\theta = -\theta\cos\theta + \int \cos\theta \, d\theta = -\theta\cos\theta + \sin\theta + C$.

   So, $\int \theta^2 \cos\theta \, d\theta = \theta^2 \sin\theta + 2\theta\cos\theta - 2\sin\theta + C$.

3. (4pts) Evaluate $\int \sin^4\theta \, d\theta$.

   First, note that

$$\begin{aligned}\sin^4\theta &= \left(\frac{1-\cos(2\theta)}{2}\right)^2 \\ &= \frac{1}{4}(1 - 2\cos(2\theta) + \cos^2(2\theta)) \\ &= \frac{1}{4}\left(1 - 2\cos(2\theta) + \frac{1+\cos(4\theta)}{2}\right).\end{aligned}$$

   So, $\int \sin^4\theta \, d\theta = \frac{1}{4}\left[\theta - \sin(2\theta) + \frac{\theta}{2} + \frac{1}{8}\sin(4\theta)\right] + C$.