Math 1B, Quiz 1 Section 203 GSI Carter

Solutions

1. (3pts) Evaluate $\int x^2 e^x \, dx$.

   With $u = x^2$ and $dv = e^x \, dx$, $du = 2x \, dx$ and $v = e^x$. So, $\int_x^2 e^x \, dx = x^2 e^x - 2\int xe^x \, dx$. Then with $u = x$ and $v = e^x \, dx$, $du = dx$ and $v = e^x$, so $\int xe^x \, dx = xe^x - \int e^x \, dx = xe^x - e^x + C$. So, $\int x^2 e^x \, dx = x^2 e^x - 2xe^x + 2e^x + C$.

2. (3pts) Evaluate $\int \theta \sin^2 \theta \, d\theta$.

   First, note that $\theta \sin^2 \theta = \frac{1}{2}\theta(1 - \cos(2\theta))$. Letting $u = \theta$ and $dv = (1 - \cos(2\theta)) \, d\theta$, we have that $du = d\theta$ and $v = \theta - \frac{1}{2}\sin(2\theta)$. So,

$$
\begin{aligned}
\frac{1}{2}\int \theta(1 - \cos(2\theta)) \, d\theta &= \frac{1}{2}\left[\theta^2 - \frac{1}{2}\theta\sin(2\theta) - \int \theta - \frac{1}{2}\sin(2\theta) \, d\theta\right] \\
&= \frac{1}{2}\left[\theta^2 - \frac{1}{2}\theta\sin(2\theta) - \frac{1}{2}\theta^2 - \frac{1}{4}\cos(2\theta) + C\right] \\
&= \frac{1}{4}\theta^2 - \frac{1}{4}\theta\sin(2\theta) - \frac{1}{8}\cos(2\theta) + C.
\end{aligned}
$$

3. (4pts) Evaluate $\int \cos^3 \theta \, d\theta$.

   Letting $u = \sin\theta$,

$$
\begin{aligned}
\int \cos^3 \theta \, d\theta &= \int (1 - \sin^2 \theta)\cos\theta \, d\theta \\
&= \int 1 - u^2 \, du \\
&= u - \frac{u^3}{3} + C \\
&= \sin\theta - \frac{\sin^3 \theta}{3} + C.
\end{aligned}
$$