

1. (3pts) Give the general solution for the differential equation $y'' + y' - 6y = 0$.

$$0 = r^2 + r - 6 = (r + 3)(r - 2)$$

$$y = c_1 e^{-3x} + c_2 e^{2x}$$

2. (3pts) Give the general solution for the differential equation $y'' - 8y' + 16y = 0$.

$$0 = r^2 - 8r + 16 = (r - 4)^2$$

$$y = c_1 e^{4x} + c_2 x e^{4x}$$

3. (4pts) Solve the following initial value problem.

$$y'' + 2y' + 2y = 0$$

$$y(0) = 0$$

$$y'(0) = 4$$

The characteristic equation is,

$$0 = r^2 + 2r + 2 = (r + 1)^2 + 1 = (r + 1 + i)(r + 1 - i).$$

Therefore $\alpha = -1$ and $\beta = 1$. So, the general solution is

$$y = e^{-x}(c_1 \cos x + c_2 \sin x).$$

Since $y(0) = 0$, $c_1 = 0$. Therefore, $y = c_2 e^{-x} \sin x$. Since $y'(0) = 4$ and $y' = c_2 e^{-x}(\cos x - \sin x)$, $c_2 = 4$. So, the solution to the initial value problem is

$$y = 4e^{-x} \sin x.$$