

1. (3pts) Give the general solution for the differential equation  $y'' + 6y' + 9y = 0$ .

$$0 = r^2 + 6r + 9 = (r + 3)^2$$

$$y = c_1 e^{-3x} + c_2 x e^{-3x}$$

2. (3pts) Give the general solution for the differential equation  $y'' + 2y' + 2y = 0$ .

The characteristic equation is

$$0 = r^2 + 2r + 2 = (r + 1)^2 + 1 = (r + 1 + i)(r + 1 - i),$$

so that  $\alpha = -1$  and  $\beta = 1$ . So, the general solution is

$$y = e^{-x}(c_1 \cos x + c_2 \sin x).$$

3. (4pts) Solve the following initial value problem.

$$y'' - 4y = 0$$

$$y(0) = 0$$

$$y'(0) = 4$$

The characteristic equation is

$$0 = r^2 - 4 = (r + 2)(r - 2),$$

so the general solution is

$$y = c_1 e^{2x} + c_2 e^{-2x}.$$

Since  $y(0) = 0$ ,  $c_1 + c_2 = 0$ . Since  $y'(0) = 4$  and  $y' = 2c_1 e^{2x} - 2c_2 e^{-2x}$ ,  $4 = 2c_1 - 2c_2$ . Therefore,  $c_1 = 1$  and  $c_2 = -1$ , so the solution to the initial value problem is

$$y = e^{2x} - e^{-2x}.$$