

# QUALIFYING EXAM SYLLABUS

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## 1. MAJOR TOPIC: COMMUTATIVE ALGEBRA (ALGEBRA)

Localization, tensor products, prime avoidance, Nakayama's lemma, associated primes and primary decomposition, integral dependence, going-up and going-down theorems, normal domains, flat modules, local criterion for flatness, complete local rings, Hensel's lemma, Hilbert's Nullstellensatz, Krull dimension, Noether normalization, Hilbert-Samuel polynomials, elimination theory, Gröbner bases, Buchberger's algorithm.

References: Eisenbud, *Commutative Algebra with a View Toward Algebraic Geometry* Chapters 2-4, 6-9, 12-15

## 2. MAJOR TOPIC: ALGEBRAIC GEOMETRY (ALGEBRA)

Schemes: affine, reduced, irreducible, regular, Noetherian, quasi-compact. properties of morphisms: finite, finite type, closed and open immersions, separated, proper, projective, affine, quasi-compact. Sheaves of modules: quasi-coherent, coherent, locally free, invertible, ample, very ample. Divisors: Weil divisors, Cartier divisors, Picard group. Cohomology of sheaves. Grothendieck vanishing theorem (statement). Serre's criterion for affineness. Čech cohomology. Cohomology of projective space.

References: Hartshorne, *Algebraic Geometry* Sections II.1-6 II.7 pp. 149-156, II.8, III.1-III.5.

## 3. MINOR TOPIC: COMPUTATIONAL BIOLOGY (APPLIED MATH)

Parametric statistical models, linear and toric models, maximum likelihood estimation, EM algorithm, implicitization of statistical models, Markov models, tree models, graphical models, tropical arithmetic, sequence alignment, Viterbi algorithm, trees and metrics, phylogenetic trees, neighbor-joining algorithm, splits equivalence theorem, 4-point condition, cherry-picking theorem, statistical models for mutation, Jukes-Cantor model.

References: Pachter and Sturmfels, ed., *Algebraic Statistics for Computational Biology* Chapters 1-4.