Various Talk Abstracts
David Penneys, UC Berkeley, 2007-present

Date: 1/8/2011
Seminar: AMS Joint Meetings, von Neumann algebras session
Organizer(s): Richard Burstein and Remus Nicoara
Title:
Abstract:

In a series of papers by Vaughan Jones, Scott Morrison, the speaker, Emily Peters, Noah Snyder, and James Tener, we implement obstructions to principal graphs in the classification of subfactors up to index 5. In part 2 (arXiv:1007.2240), we eliminate two problematic "weeds" via a quadratic tangles test developed by Jones (arXiv:1007.1158), and in part 4 (in preparation), we eliminate vines via a theorem about cyclotomicity of graph norms by Frank Calegari, Morrison, and Snyder (arXiv:1004.0665).

Date: 9/17/2010
Seminar: Subfactor
Organizer(s): David Penneys
Title: An infinite index subfactor with finite dimensional relative commutants
Abstract:

After a brief introduction to infinite index subfactors, we will focus on an example suggested by Jones which has finite dimensional relative commutants, the proof of which is joint with Makoto Yamashita (U. Tokyo) and Steven Deprez (KU Leuven). This example differs from the finite index case in many surprising ways.

Date: 9/10/2010
Seminar: Student Subfactor
Organizer(s): Michael Hartglass and James Tener
Title: I've computed the principal graph of this subfactor. Now what?
Abstract:

A powerful invariant of a subfactor is its planar algebra (or paragroup or λ-lattice). A powerful invaraint of the (subfactor) planar algebra is its principal graph. All subfactors of "small index" (besides Temperley-Lieb), where small means (probably) below 5 (and hopefully below $3+\sqrt{5}$), are finite depth, where the story is well understood for subfactors of the hyperfinite II_1-factor via Popa's reconstruction theorem. However, for infinite depth, it is unclear what to do next.

We will describe some constructions which help classify infinite depth subfactors, including Ocneanu's asymptotic inclusion, the Longo-Rehren construction, and Popa's symmetric enveloping algebra.

Date: 7/26/2010
Seminar: Kyushu University Operator Algebras
Organizer(s): Yasuo Watatani and Toshihiko Masuda
Title: Elimintating weeds with annular multiplicities *10 via quadratic tangles
Abstract: Same as below.

Date: 7/8/2010
Seminar: University of Tokyo Operator Algebras

Organizer(s): Yasuyuki Kawahigashi
Title: Eliminating weeds with annular multiplicities *10 via quadratic tangles
Abstract:

In recent work with Morrison, Peters, and Snyder, we eliminate two families of possible principal graphs with graph norms less than 5 using Jones' quadratic tangles improvement of Ocneanu's triple point obstruction.

Date: 5/11/2010
NCGOA symposium Vanderbilt University
Organizer(s): Dietmar Bisch, Alain Connes, Jesse Peterson, Gennadi Kasparov, and Guoliang Yu
Title: Embedding subfactor planar algebras in graph planar algebras
Abstract:

Abstract: In her thesis, Peters found the Haagerup subfactor planar algebra inside the graph planar algebra of its principal graph. Bigelow, Morrison, Peters, and Snyder used this technique to construct the extended Haagerup subfactor. In applying this technique, one assumes (but does not rely on) the fact that a finite depth subfactor planar algebra embeds in the graph planar algebra of its principal graph. We give one proof of this fact due to V. Jones and the speaker.

Date: 4/5/2010
Seminar: Quantum Geometry
Organizer(s): Marc Rieffel and Matt Tucker-Simmons
Title: Categorified Morita equivalence
Abstract:

After a quick reminder about Morita equivalence for rings, we will briefly discuss how a fusion category "categorifies" the notion of a ring. We will then discuss Müger's definition of Morita equivalence for fusion categories and how it gives rise to subfactors.

Date: 3/19/2010
Seminar: Student Sufactor
Organizer(s): Michael Hartglass and James Tener
Title: Lattices and quadrilaterals of factors
Abstract:

Let $\mathcal{L}$ be a finite lattice of II_1-factors. We denote

$$M = \bigvee_{L\in\mathcal{L}} L \text{ and } N = \bigwedge_{L\in\mathcal{L}} L.$$

We will show (sketch) that $\mathcal{L}$ gives a multishaded planar algebra when $[M : N] < \infty$. Focusing on the case when $\mathcal{L}$ is a quadrilateral

$$N = P \wedge Q \subset P, Q \subset P \vee Q = M,$$

we will discuss notions of commuting, cocommuting, and irreducibility and relate these to simplification of mutlishaded planar diagrams.

Date: 2/5/2010
Seminar: GRASP (Geometry, Representation Theory, and some Physics)
Organizer(s): Theo Johnson-Freyd and Harold Williams

Title: What is a subfactor, and why should I care?
Abstract:

By deep theorems of Jones and Popa, (suitably nice) subfactors are equivalent to (suitably nice) planar algebras which are, in turn, equivalent to (suitably nice) unitary 2-categories with 2 objects (or “bi-oidal” categories). I will give a brief definition of each of the above, after which I will briefly try to persuade you that you should care.

Date: 1/29/2010
Seminar: Student Subfactor
Organizer(s): Michael Hartglass and James Tener
Title: Quadratic tangles part I
Abstract:

Suppose you meet a planar algebra walking down the street. Whats the first and second question you would ask it? If youve met many a planar algebra before, youd probably ask the following two:

(1) What do Temperley-Lieb tangles look like to you? (What is the image of the Temperley-Lieb tangles in said planar algebra?)

(2) What are your irreducible annular Temperley-Lieb (ATL) modules?

These questions correspond to the fact that the easiest tangles to deal with are those with no input disks, and the next easiest tangles are those with one input disk. Quadratic tangles, which have exactly two input disks, are the next in line, and the situation gets a lot more complicated. This will be the first in a series of talks on applying quadratic tangle techniques to get to know your new planar algebra friend.

Date: 12/11/2009
Seminar: Student Subfactor
Organizer(s): David Penneys
Title: The Details of the Embedding Theorem, part II
Abstract:

We will discuss some details behind the proof of the embedding theorem given on 11/20 that were either omitted on 12/4 or slightly incorrect. In particular, we will present the corrected Pimsner-Popa basis!

Date: 12/4/2009
Seminar: Student Subfactor
Organizer(s): David Penneys
Title: The Details of the Embedding Theorem
Abstract:

We will discuss some omitted details behind the proof of the embedding theorem given on 11/20. Much of the talk will be focused on loop algebras, which are a reformulation of Ocneanu and Sunder’s path algebras with a more “GNS” rather than “spatial” flavor.

Date: 11/20/2009
Seminar: Subfactor
Organizer(s): David Penneys
Title: Another Proof of the Embedding Theorem
Abstract:

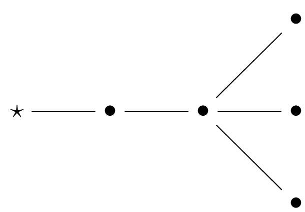
We will give another proof due to Jones that a finite depth, subfactor planar algebra is embedded in the graph planar algebra of its principal graph.

Date: 11/13/2009
Seminar: Student Subfactor
Organizer(s): David Penneys
Title: Fusion Categories
Abstract:

This will be more of an informal discussion on subfactors, planar algebras, and fusion categories.

Date: 10/2/2009
Seminar: Subfactor
Organizer(s): David Penneys
Title: The Representation Theory of ATL, part II
Abstract:

An annular Temperley-Lieb module is a functor $F\colon ATL \to Hilb_{f.d.}$, where ATL is the annular Temperley-Lieb category and $Hilb_{f.d.}$ is the category of finite dimensional complex Hilbert spaces. First, we will finish describing the irreducible ATL-modules for $\delta > 2$. We will then decompose the planar algebra of the bipartite graph



into irreducible ATL-modules.

Date: 9/18/2009
Seminar: Subfactor
Organizer(s): David Penneys
Title: The Representation Theory of ATL
Abstract: The annular Temperley-Lieb algebras may be considered together as a category, which we will denote ATL. Objects are $2n$ points on a circle, morphisms are annular shaded planar tangles, and composition is insertion of tangles (using some $*$s to line them up appropriately). We will discuss the Hilbert space representations of ATL, i.e., functors $F : ATL?Hilb_{f.d.}$, where $Hilb_{f.d.}$ is the category of finite dimensional complex Hilbert spaces together with bounded maps.

As a C^*-planar algebra with an invariant inner product is a Hilbert ATL-module, we may decompose it into its irreducible ATL-submodules. This technique is exploited by Peters to construct the Haagerup subfactor, an exotic, finite depth subfactor of small index, from a graph planar algebra. This technique is also the first step in the classification program of Jones, Morrison, Peters, and Snyder who hope to achieve classification of such subfactors up to index 5.

Date: 9/11/2009
Seminar: Student Subfactor
Organizer(s): David Penneys
Title: Introduction to Planar Algebras, part II
Abstract:

Last time we defined the notion of a planar algebra, and we gave the example of the Temperley-Lieb planar algebra. This time, we will give more complicated examples, including the planar algebra of a bipartite graph. We will also discuss how planar algebras are modules over the annular category, and we will show how planar algebras are completely decomposable into irreducible annular modules. We may then give some examples of annular modules which do not behave nearly as well as those arising from planar algebras.

Date: 9/4/2009
Seminar: Subfactor
Organizer(s): David Penneys
Title: Introduction to Planar Algebras, part I
Abstract:

Jones showed that the standard invariant of a finite index II_1-subfactor forms a planar algebra. Popa, and later Ocneanu, first showed how to reconstruct subfactors from the combinatorial data of the standard invariant. Much later, Jones, Guionnet, Shlyakhtenko, and Walker (and also Kodiyalam and Sunder) described this process from the planar algebra viewpoint.

In these two talks, we will define planar algebras and give many examples. We will then discuss how they are modules over the annular category, which is the first step in the program of Jones, Morrison, Peters, and Snyder (at Berkeley, we refer to them as Vaughan, Scott, Emily, and Noah respectively) to classify exotic, finite depth subfactors of small index (less than 5).

This first talk is meant to be very accessible as it is an introduction. We will assume our audience is familiar with vector spaces and tensor products.

Date: 4/10/2009
Seminar: Student Subfactor
Organizer(s): Jose Alameida and David Penneys
Title: Properties of Groups and von Neumann Algebras, part II
Abstract:

This week we will go over some alternate definitions of property (T) for groups and show their equivalence. We will then discuss relative property (T). Finally, we will begin discussing how these properties (amenability, the Haagerup property, and property (T)) should be defined for von Neumann algebras.

Date: 4/3/2009
Seminar: Student Subfactor
Organizer(s): Jose Alameida and David Penneys
Title: Properties of Groups and von Neumann Algebras, part I
Abstract:

Over the course of several talks, we will discuss properties that countable discrete groups and (finite) von Neumann algebras can satisfy, such as amenability, the Haagerup property, aka (H), and property (T). One first defines these properties for a group Γ, then for the group von Neumann algebra $L\Gamma$, and finally for an arbitrary von Neumann algebra. We will give several definitions of some of these properties, in particular via affine actions of Γ on Hilbert spaces using cocycles, as this viewpoint emphasizes the difference between (H) and (T).

Date: 3/20/2009
Seminar: Subfactor
Organizer(s): David Penneys

Title: A Presentation of the Annular Temperley-Lieb Category, Part II
Abstract:

We will begin this second talk by defining an abstract category $\mathsf{a}\Delta$ via generators and relations based on the distinguished tangles we discussed last time. We will then give some unique decomposition theorems for tangles in the annular Temperley-Lieb category Atl, and we will define and prove the existence of a standard form for words in $\mathsf{a}\Delta$. Using these main ingredients, we will cook up the promised isomorphism of involutive categories $\mathsf{a}\Delta \cong \mathsf{Atl}$.

Date: 3/13/2009
Seminar: Subfactor
Organizer(s): David Penneys
Title: A Presentation of the Annular Temperley-Lieb Category, Part I
Abstract:

This will be the first in a series of two or three talks in which we give a presentation of the annular Temperley-Lieb category via generators and relations. In this introductory talk, we will begin by discussing the semi-simplicial, simplicial, and cyclic categories. We will present these caegories via generators and relations, and we will describe pictorial representaitons which motivate the definition of the annular Temperley-Lieb category Atl. Keeping the cyclic category in mind, we will then offer candidates for the generators and relations of Atl. Time permitting, we will begin to discuss some unique decomposition results for annular tangles.

Date: 2/26/2009
Seminar: IMSc Colloquium
Organizer(s): Vijay Kodiyalam and V.S. Sunder
Title: Categories and Pictures
Abstract:

Throughout history, mathematics has tended toward abstraction, and some of the most abstract mathematics occurs in category theory. Pictures on the other hand are concrete, but it is quite difficult to give them precise mathematical meaning. We will present some notable categories, namely the semi-simplicial, simplicial, and cyclic categories via generators and relations, and we will give pictorial representations of such categories. We will then present a category whose morphisms are pictures, namely annular (m, n)-tangles in the sense of Jones' planar algebras, and time permitting, we will discuss some work in progress of presenting it in terms of generators and relations.

Date: 1/23/2009
Seminar: Subfactor
Organizer(s): David Penneys
Title: The Quasi Planar Algebra of an Infinite Index, Extremal II_1-Subfactor, part I
Abstract:

In his dissertation, Burns took the first step towards finding limited planar structure for infinite index II_1-subfactors. He gave the first definition of extremality, and showed it is equivalent to the existence of a rotation operator on $N' \cap L^2(M_k)$ (where $N \subset M \subset M_1 \subset \cdots$ is the basic construction). This will be the first in a series of n talks in which we give the first definitions of

(1) the suboperad of the planar operad consisting of internally shaded connected tangles,

(2) a quasi planar algebra as an algebra over a suboperad $\mathbb{O}$ of the planar operad $\mathbb{P}$, and

(3) the standard invariant of an infinite index, extremal II_1-subfactor,

after which we will show the standard invariant is a quasi planar algebra over the suboperad of internally shaded connected tangles (although i still need to work out the details on isotopy invariance).

Date: 1/23/2009
Seminar: Student Subfactor
Organizer(s): Jose Alameida and David Penneys
Title: A "subfactor" planar algebra everyone will understand
Abstract:

If you have ever wondered what a subfactor or a planar algebra is, or if you have ever wondered about the analysis behind planar algebras, this is the talk for you. From the simplest example of a subfactor, namely

$$\mathbb{C}I_2 \subset M_2(\mathbb{C}),$$

we will use the language of II_1-subfactors to construct a planar algebra (which is actually the planar algebra of the bipartite graph with two vertices and two edges connecting them). The only prerequisite knowledge assumed will be how to take tensor product of matrices. Plus, I'll give you a nice handout as a souvenir.

Date: 9/5/2008
Seminar: Subfactor
Organizer(s): Vaughan Jones
Title: Examples of Infinite Index Subfactors, part I
Abstract:

We will construct the Jones tower $N \subset M \subset M_1 \subset M_2 \subset \cdots$ for the infinite index subfactor $N = R \otimes 1 \subseteq R \otimes R = M$. Along the way, we will use some of the techniques that we discussed in the introductory talk on type II_∞-factors and infinite index subfactors.

Date: 9/5/2008
Seminar: Student Subfactor
Organizer(s): David Penneys
Title: Correspondences and Bimodules
Abstract:

Let P, Q be II_1-factors. It is well known that if H is a bifinite $P - Q$ correspondence (Hilbert space bimodule ${}_PH_Q$ with $\dim_P(H)$ and $\dim_Q(H)$ finite), then the set of left P-bounded vectors and the set of right Q-bounded vectors coincide. Denoted H^0, this set of bounded vectors turns out to be a $P-Q$ bimodule. We will explain these facts along with a recent result of Jones which says that the bifinite correspondence ${}_PH_Q$ is irreducible if and only if the bimodule ${}_P(H^0)_Q$ is irreducible.

Date: 8/29/2008
Seminar: Subfactor
Organizer(s): Vaughan Jones
Title: Introduction to II_∞-factors and Infinite Index Subfactors
Abstract:

We will compare and contrast II_1-factors and II_∞-factors by discussing traces, standard form, and commutant. We will then compare and contrast finite and infinite index subfactors $M_0 \subset M_1$ by discussing conditional expectations, operator-valued weights, the basic construction, bases, and extremality. Most of what will be covered will not be proven as the material is quite technical.

Date: 6/27/2008, 7/3/2008, 7/10/2008, 7/17/2008
Seminar: Summer Student Operator Algebras
Organizer(s): David Penneys
Title: There is Only One Hyperfinite II_1-factor
Abstract:

Defintion: A II_1-factor is called hyperfinite if it is generated by an increasing union of finite dimensional von Neumann algebras.

Defintion: A II_1-factor M is called approximately finite dimensional (AFD) if for all $x_1, \ldots, x_n \in M$ and all $\varepsilon > 0$, there exists a finite dimensional $*$-subalgebra $N \subset M$ and elements $y_1, \ldots, y_n \in N$ such that $\|x_i - y_i\|_2 \leq \varepsilon$ for all $i = 1, \ldots, n$.

We will prove the following theorem in all its splendor (or gory detail if you don't like analysis):
Theorem: Let M be a II_1-factor. The following are equivalent:
(1) M is generated by an increasing sequence $(N_n)_{n\in\mathbb{N}}$ of factors where N_n is of type I_{2^n} for all $n \in \mathbb{N}$,
(2) $M \cong \bigotimes_{i=1}^{\infty} M_2(\mathbb{C})$,
(3) M is hyperfinite, and
(4) M is generated by a countable family of elements, and M is AFD.

Date: 4/28/2008
Seminar: Quantum Geometry
Organizer(s): Marc Rieffel, Matt Tucker-Simmons, Patrick Barrow
Title: Cyclic Homology, part II
Abstract:

This week, we will give the definition of cyclic homology, and we will compute the Hochschild and cyclic homology of the most trivial k-algebra possible, namely k (where k is a unital commutative ring). We will then prove that Hochschild and cyclic homology are Morita invariant, i.e. Morita equivalent unital k-algebras have isomorphic Hochschild and cyclic homology.

Date: 4/21/2008
Seminar: Quantum Geometry
Organizer(s): Marc Rieffel, Matt Tucker-Simmons, Patrick Barrow
Title: Cyclic Homology, part I
Abstract:

This will be the first of two talks whose aim will be to compute the cyclic homology of something. In this first talk, we will give the definition of a cyclic module and give many examples. We will then define cyclic homology for cyclic modules.

Date: 4/4/2008
Seminar: Subfactor
Organizer(s): Vaughan Jones
Title: Cyclic Homology
Abstract:

Cyclic homology was first defined by Connes using the cyclic complex C_*^λ. Loday, Quillen, and/or Tsygan formulated another definition using the cyclic bicomplex CC_{**}, which we discussed last time. We will compare and contrast these definitions of cyclic homology and prove that they are the same for cyclic modules over rings that contain $\mathbb{Q}$. We will then discuss some tools for computing cyclic homology, such as an alternate cyclic bicomplex and Connes' Periodicity Exact Sequence.

Date: 3/25/2008
Seminar: GWU Analysis
Organizer(s): Inhyeop Yi, E. Arthur Robinson, Jr.
Title: Planar Algebras and Knot Polynomials
Abstract:

After Vaughan Jones initiated the study of subfactors, he came across an invariant of knots via a representation of the braid group in the Temperley-Lieb algebra, which, in turn, has representations in the hyperfinite II_1-factor. He also found that subfactors have a certain "planar" structure, which lead to the definition of a planar algebra. This talk will not be about subfactors (overtly); rather, we will define a general planar algebra over a ring, and focus on the Temperley-Lieb planar algebra as a concrete example. We will then discuss some planar algebras associated to knot polynomials, such as the Jones, Conway-Alexander, and (time permitting) HOMFLYPT polynomials.

Date: 3/21/2008
Seminar: Student Subfactor
Organizer(s): David Penneys
Title: Knot Polynomials
Abstract:

We will construct the Jones polynomial using a representation of the braid group in the Temperley-Lieb algebra and representations of the Temperley-Lieb algebra in the hyperfinite II_1 factor. We will then construct the Jones polynomial again using skein relations. If there is time left over, we will also discuss the Conway-Alexander polynomial.

Date: 3/19/2008
Seminar: Many Cheerful Facts
Organizer(s): MGSA
Title: What Everyone Should Know About Inner Products
Abstract:

If I had to pick a favorite function, it would be an inner product. Actually, it might be the trace on a II_1 factor, but if I gave a talk about that, not many people would show up. Plus, the trace gives an inner product, but that's not on the quiz at the end, so you don't have to know that.

We will briefly discuss the merits of an inner product, e.g. it gives a norm in which balls are round, and it gives a canonical identification of H with H^*. We will then solve the fundamental problem of the inner product: is it linear on the left or on the right? (In case you were wondering, the answer is: "it should depend on the following notation:

$$\langle x, y\rangle = \langle y|x\rangle$$

where the left one is linear on the left, and the right one is linear on the right." Now you don't even have to show up!) The solution will segue nicely into a discussion on Dirac notation, which will ultimately meander into the world of C^*-valued inner products. Actually, we won't be meandering whatsoever since I will have led us there all along.

Date: 3/14/2008
Seminar: Subfactor
Organizer(s): Vaughan Jones
Title: Cyclic Homology and General Planar Algebras over Rings
Abstract:

Given a cyclic object in an abelian category, one can define its Hochschild and cyclic homologies. We will discuss these constructions and focus on the example provided by a general planar algebra over a ring, which is a cyclic module in two different ways. In particular, we will compute some homology groups for the Temperley-Lieb general planar algebra over $\mathbb{Z}$ with $\delta = 0$ and the general planar algebra associated to the Conway-Alexander polynomial (which also has $\delta = 0$).

Date: 2/15/2008
Seminar: Student Subfactor
Organizer(s): David Penneys
Title: Relative Tensor Product and Fusion Algebras
Abstract:

If A, B, C are rings and ${}_AH_B, {}_BK_C$ are bimodules, then we are all familiar with how to define the tensor product ${}_AH \otimes_B K_C$. When M, N, P are II_1-factors and ${}_MH_N, {}_NK_P$ are bimodules (with finite index as usual), then the process, Connes' relative tensor product, is a little more tricky. We will discuss the relative tensor product of bimodules and pay close attention to the case where $N \subset M$ are subfactors of finite index and H and K are $L^2(M)$ with various N, M actions. This analysis together with a noncommutative Radon-Nikodym theorem and a theorem characterizing normal maps makes this construction far less opaque. Time permitting, we will then discuss the fusion algebra associated to $N \subset M$.

Date: 12/3/2007, 12/10/2007
Seminar: Student Noncommutative Geometry
Organizer(s): Marc Rieffel, Matt Tucker-Simmons, and Patrick Barrow
Title: An Introduction to II_1 (Sub)Factors and Bimodules
Abstract:

We give a brief introduction to II_1 (sub)factors including the coupling constant, the index, and the basic construction. We will then discuss bimodules and the relative tensor product. Along the way, familiar faces will appear, such as II_1 factor-valued pairings coming from a noncommmutative Radon-Nikodym Theorem.

Date: 9/12/2007
Seminar: Many Cheerful Facts
Organizer(s): MGSA
Title: Semidirect Product : Groups :: Group Mesaure Space Construction : Von Neumann Algebras
Abstract:

Abelian Von Neumann algebras acting on a separable Hilbert spaces are boring. Each one is isometrically-*-isomorphic to $L^\infty(X, \mu)$ for some $X \subseteq \mathbb{C}$. If μ is a finite, regular Borel measure on a locally compact Hausdorff space X, a group G of bijections of X can be considered as a group of automorphisms of the abelian Von Neumann algebra $L^\infty(X, \mu)$ in a natural way. If the group acts nontrivially, we can use the group measure space consruction as a recipe for creating the nonabelian Von Neumann algebra $L^\infty(X, \mu) \rtimes G$

from $L^\infty(X, \mu)$, much like the semi-direct product of groups constucts a nonabelian group from abelian ones. This is most excellent as it cooks up a myriad of nontrivial examples of nonabelian Von Neumann algebras, even factors.