Math 252 - Exercises XV
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(**5**) Show that the Feit-Thompson Theorem ("odd groups are solvable") is equivalent to the statement that finite nonabelian simple groups have even order.

**Solution**: Before we begin, notice that the statement that finite nonabelian simple groups have even order is equivalent to the statement that all finite simple groups of odd order are abelian.

First suppose that the Feit-Thompson Theorem holds, and let $G$ be a simple group of odd order. Then $G$ must be solvable. But the only composition factor of a composition series for $G$ is $G$ itself, so $G$ must be abelian.

Conversely, suppose that finite nonabelian simple groups have even order, and let $G$ be a finite group of odd order. Then in any composition series of $G$, the composition factors all have odd order and therefore are abelian. It follows that $G$ is solvable, so the Feit-Thompson Theorem holds.