8. Let $G = K \rtimes H$ be a Frobenius group in the usual notation. If $|H|$ is even, show that K is abelian.

Proof. Let $h \in H$ be an element of order 2, and let $\varphi : K \to K$ be the bijective map defined in exercise 7 ($\varphi(x) = x^{-1}x^h$). The conjugation action of h on K defines an automorphism of K. We have

$$\varphi(x)^h = hx^{-1}hx = \varphi(x)^{-1},$$

and since φ is bijective it follows that the automorphism of K defined by h is $g \mapsto g^{-1}$. Clearly this map is an automorphism iff K is abelian.

□