Math 252 Assignment 16

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(6) Let $\overline{\chi} \in \mathrm{Irr}(\overline{G})$ where $\overline{G}$ is a factor group of G. If $\chi \in \mathrm{Irr}(G)$ is the "pullback" of $\overline{\chi}$ to G, show that $s(\chi) = s(\overline{\chi})$.

Solution. Let $H \triangleleft G$ such that $\overline{G} = G/H$. Then $\chi = \overline{\chi} \circ \varphi$ where $\varphi\colon G \to G/H$ is the canonical epimorphism. Note that for each $gH \in G/H$, the same number of elements in G map to gH, namely $|H|$. Moreover, since $(gH)^2 = g^2H$, we have that $\chi(g^2) = \overline{\chi}(g^2H) = \overline{\chi}((gH)^2)$. Hence,

$$s(\chi) = \frac{1}{|G|}\sum_{g\in G}\chi(g^2) = \frac{1}{|G|}|H|\sum_{gH\in G/H}\overline{\chi}((gH)^2) = \frac{1}{[G\colon H]}\sum_{gH\in G/H}\overline{\chi}((gH)^2) = s(\overline{\chi}).$$

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