Math 252: Exercises XIX by Dmitry Pavlov, pavlov@math.

Problem 1. Let G be the n-dimensional affine group over q-element field that acts naturally on the n-dimensional vector space E. Show that this is a Frobenius action if and only if $n = 1$ and $q > 2$.

Solution. Note that all translations do not have any fixed points, hence they are inside Frobenius kernel K. If $n > 1$, then we have an obvious example of invertible affine transformation with more than one fixed point that is different from identity. Indeed, let A exchange the first two coordinates. Obviously, this transformation is its own inverse, it is different from identity and it has more than one fixed point (all points whose first two coordinates are equal are fixed points). Hence the action is not Frobenius. If $n = 1$ and $ax + b = x$, then $(a - 1)x = -b$ and $x = -b(a - 1)^{-1}$ if $a \neq 1$. Hence we have exactly one fixed point when $a \neq 1$. And the case $a = 1$ corresponds to translations, which belong to Frobenius kernel. Hence $|K| = |E|$ and the action is Frobenius except when $q = 2$, when we have $|E| = |G|$ and the action is not Frobenius.