# Math 252: Representation Theory

## Exercises XVIII

**Problem 5.** *Let $N \subseteq H \subseteq G$ be groups, with $N \triangleleft G$. If $N$ acts trivially on a $kH$-module $V$, show that $N$ also acts trivially on the induced module $V^G$.*

*Proof.* Recall the induced module is $V^G = kG \otimes_{kH} V$. Let $n \in N$ and $g \otimes v \in V^G$. We calculate:

$$\begin{aligned} n \cdot (g \otimes v) &= ng \otimes v \\ &= gn' \otimes v && \text{(where } g^{-1}ng = n' \in N\text{)} \\ &= g \otimes n'v \\ &= g \otimes v && \text{(since } N \text{ acts trivially on } V\text{)}. \end{aligned}$$

We conclude that $N$ acts trivially on the induced module $V^G$. □

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