Math 252: Representation Theory

Exercises XIV

Problem 1. *Let $G = \langle h | h^4 = 1 \rangle$ act on $V = kx \oplus ky$ by $h \cdot x = y$ and $h \cdot y = -x$. Use Molien's Formula to compute $P_{S(V)^G}$.*

Solution. Observe that h acts on V by multiplication by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. We compute the following:

$$\begin{aligned}
\det(I - 1 \cdot t) &= \det \begin{pmatrix} 1-t & 0 \\ 0 & 1-t \end{pmatrix} = (1-t)^2 \\
\det(I - h \cdot t) &= \det \begin{pmatrix} 1 & t \\ -t & 1 \end{pmatrix} = 1 + t^2 \\
\det(I - h^2 \cdot t) &= \det \begin{pmatrix} 1+t & 0 \\ 0 & 1+t \end{pmatrix} = (1+t)^2 \\
\det(I - h^3 \cdot t) &= \det \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix} = 1 + t^2.
\end{aligned}$$

Using Molien's Formula, we get

$$\begin{aligned}
P_{S(V)^G}(t) &= \frac{1}{|G|} \sum_{g \in G} \frac{1}{\det(I - g \cdot t)} \\
&= \frac{1}{4} \left(\frac{1}{(1-t)^2} + \frac{1}{1+t^2} + \frac{1}{(1+t)^2} + \frac{1}{1+t^2} \right) \\
&= \frac{1+t^4}{(1-t^2)(1-t^4)} \\
&= 1 + t^2 + 3t^4 + 3t^6 + 5t^8 + 5t^{10} + 7t^{12} + \cdots
\end{aligned}$$

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