Exercise 7: Does the conclusion of Exercise 6 hold if H is abelian but G is not?

Solution: The result may not hold. For example, choose $G = \mathbb{D}_4$ and $H = \{1, r^2\} \lhd G$. G is nonabelian whereas H is abelian. As a character, pick $\chi = (2, -2, 0, 0, 0)$, corresponding to the 2-dimensional representation over $\mathbb{R}$ (which is still irreducible over $\mathbb{C}$). In this case $\dim U = \chi(1) = 2$, and $\chi(G \setminus H) = 0$. However, $[G : H] = \frac{8}{2} = 4 \nmid 2 = \dim U$. $\square$