Exercise 5: Show that the Feit-Thompson Theorem ("Odd Groups are Solvable") is equivalent to the statement that finite nonabelian simple groups have even order.

Solution:
($\Rightarrow$) Suppose we know that odd groups are solvable. Let G be a finite nonabelian simple group. Assume G has odd order, then G is solvable and the corresponding series is given by the derived series. Hence $\{1\} \subsetneq [G : G] \subsetneq G$ (G is nonabelian), contradicting the simplicity of G.

($\Leftarrow$) We will prove the result by induction (on odd positive integers). The result for $|G| = 1$ is clear. Let $n \in \mathbb{N}$ odd integer and G a group of odd order n, and assume the result is true for all odd integers $k < n$. If G is abelian, it is solvable. Thus, we can assume G is nonabelian. Thus, by hypothesis G can't be simple, and so there exists $\{1\} \subsetneq H \subsetneq G$, $H \lhd G$. In particular G/H and H are groups of odd order (by Lagrange's Thm) with $|G/H|, |H| < n$, so by inductive hypothesis H and G/H are solvable groups. Hence, G is also solvable. $\square$