Math 252 Exercises XV #5

James Cook

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Exercise

Show that the Feit-Thomson Theorem

$$\text{Odd groups are solvable.} \tag{0}$$

is equivalent to the statement

$$\text{Finite nonabelian simple groups have even order.} \tag{1}$$

Solution

$\Rightarrow$

Assume (0) and let G be a finite nonabelian simple group. The only proper normal subgroup of G is $\{1\}$, and $G/\{1\}$ is not abelian, so G is not solvable, so G has even order.

$\Leftarrow$

Assume (1), and let G be a group of odd order n. Assume by strong induction that every group of odd order less than n is solvable.

If G is abelian, it is solvable. If G is not abelian, then by (1) G has a nontrivial normal subgroup H. By the induction hypothesis, H and G/H are solvable, and so their solutions can be combined to a solution of G.