

## MATH 252 EXERCISES XVI PROBLEM 6

DAVID BERLEKAMP

### 1. PROBLEM:

Let  $\bar{\chi} \in \text{Irr}(\bar{G})$ , where  $\bar{G}$  is a factor group of  $G$ . If  $\chi \in \text{Irr}(G)$  is the "pullback" of  $\bar{\chi}$  to  $G$ , show that  $s(\chi) = s(\bar{\chi})$ .

### 2. SOLUTION:

Note that the preimages of each  $\bar{g} \in \bar{G}$  are the same size in  $G$ , namely the cardinality of the kernel of the canonical map  $\phi : G \rightarrow \bar{G}$ ,  $|G|/|\bar{G}|$ . On each such preimage,  $\chi$  takes a constant value (the value of  $\bar{\chi}$  on  $\bar{g}$ ). Thus we have

$$\begin{aligned} |G|s(\chi) &= \sum_{g \in G} \chi(g^2) = \sum_{\bar{g} \in \bar{G}} \sum_{g \in \phi^{-1}(\bar{g})} \chi(g^2) \\ &= \sum_{\bar{g} \in \bar{G}} \sum_{g \in \phi^{-1}(\bar{g})} \chi(\bar{g}^2) = |\ker(\phi)| \sum_{\bar{g} \in \bar{G}} \chi(\bar{g}^2) = |\ker(\phi)| |\bar{G}| s(\bar{\chi}) \end{aligned}$$

and as  $|\ker(\phi)| = |G|/|\bar{G}|$ , we are done.