MATH 252 EXERCISES XVI PROBLEM 4

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1. Problem:

Improving upon Ex. XV. 5, show that Feit-Thompson implies that the order of a nonabelian (finite) simple group is divisible by 4.

2. Solution:

By Feit-Thompson, a nonabelian finite simple group G has even order. If its order is $2n$ where n odd, consider G as a subgroup of S_{2n}. Then any involution u is a product of n transpositions; as n odd, the subgroup of G consisting of even permutations is proper, nontrivial of index 2 (lest G be abelian), and normal, contradicting simplicity of G. Therefore $|G|$ is in fact divisible by 4.

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