MATH 252 EXERCISES XV PROBLEM 5

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1. Problem:

Show that the Feit-Thompson Theorem ("odd groups are solvable") is equivalent to the statement that finite nonabelian simple groups have even order.

2. Solution:

A finite nonabelian simple group is not solvable, so the Feit-Thompson Theorem implies that it must have even order. Conversely, assume all finite nonabelian simple groups have even order, and let G have odd order. Then the composition factors in any composition series of G have size dividing the order of G, hence are odd; by assumption they are then abelian, so G is solvable.

Date: Nov. 2007.