

## MATH 252 EXERCISES XIV PROBLEM 1

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### 1. PROBLEM:

Let  $G = \langle h \rangle$  ( $o(h) = 4$ ) act on  $V = kx \oplus ky$  by  $hx = y$  and  $hy = -x$ . Use Molien's formula to compute  $P_{S(V)^G}$ .

### 2. SOLUTION:

We have  $\det(\mathbf{I} - ht) = \det(\mathbf{I} - h^3t) = 1 + t^2$ , while  $\det(\mathbf{I} - h^0t) = \det((1 - t)\mathbf{I}) = (1 - t)^2$ , and  $\det(\mathbf{I} - h^2t) = \det((1 + t)\mathbf{I}) = (1 + t)^2$ . Hence Molien's formula gives

$$\begin{aligned} P_{S(V)^G}(t) &= \frac{1}{|G|} \sum_{g \in G} \det(\mathbf{I} - gt)^{-1} \\ &= \frac{1}{4} ((1 - t)^{-2} + (1 + t)^{-2} + 2(1 + t^2)^{-1}) = \sum_{n \geq 0} t^{4n} + nt^{2n}. \end{aligned}$$