QUALIFYING EXAMINATION FOR DIOGO OLIVEIRA E SILVA

July 9, 2008, 11am

Michael Christ, Marc Rieffel (chair), Michael Hutchings, Noureddine El Karoui (Statistics).

MAJOR TOPIC: HARMONIC ANALYSIS (CLASSICAL ANALYSIS)

Basic Topics. Convolution. Approximate identities. The inversion formula and Plancherel's theorem. The Schwartz space. Temperate distributions. Poisson summation formula.

Fourier Series. L^2 convergence. Decay of Fourier coefficients. Pointwise and uniform convergence of Fourier series. Riemann's localization theorem. Cèsaro means and Fejér's theorem. Norm convergence. Wiener's Tauberian theorem.

Applications. Weyl's equidistribution theorem. Minkowski's theorem. Evaluation of certain infinite series.

Hardy-Littlewood Maximal Function. Vitali covering lemma and weak type $(1, 1)$ estimate. Lebesgue differentiation theorem.

Interpolation. Riesz-Thörin theorem. Marcinkiewicz interpolation theorem. Applications: Hausdorff-Young theorem and Young's convolution inequality.

Singular Integral Operators. Calderón-Zygmund decomposition. L^p boundedness of Calderón-Zygmund operators. Homogenous distributions. Almost everywhere existence of principal-value integrals.

Almost-Orthogonality and Littlewood-Paley Theory. Almost-orthogonality in Hilbert spaces. Littlewood-Paley theory. Converse inequalities. Application: convergence of lacunary partial sums of Fourier series.

References:

1. M. Christ, *Euclidean Harmonic Analysis Notes for Mathematics 258.*
2. Stein and Shakarchi, *Fourier Analysis - An Introduction.*

MAJOR TOPIC: BANACH ALGEBRAS AND SPECTRAL THEORY (MODERN ANALYSIS)

Banach Spaces. Baire category theorem. Open mapping and closed graph theorems. Banach-Steinhaus theorem. Hahn-Banach theorem.

Hilbert Spaces. Cauchy-Schwarz inequality. Riesz representation theorem for linear functionals.

Operators on Hilbert Spaces. Topologies on $\mathcal{B}(\mathcal{H})$. Orthogonal projections. Polar decomposition. Compact, trace-class and Hilbert-Schmidt operators. Fredholm operators and the Calkin algebra. Normal operators and the continuous functional calculus. The spectral theorem for self-adjoint elements. Examples: multiplication operators, the shift operator.

Banach Algebras. Spectrum of an element. Spectral radius formula. Spectral mapping. Gelfand transform. Examples: $C(X)$, $L^1(\mathbb{R})$ and the Wiener algebra.

C^*-Algebras. Adjoining an identity. Spectral permanence. Commutative C^*-algebras. Gelfand-Naimark theorem. Approximate identities. Ideals and quotients. States and the GNS construction.

References:

1. W. Rudin, *Real and Complex Analysis* (chapters 4 and 5).
2. W. Arveson, *A Short Course on Spectral Theory.*
3. P. Halmos, *What Does the Spectral Theorem Say?*.

MINOR TOPIC: ALGEBRAIC TOPOLOGY (GEOMETRY)

The Fundamental Group. $\pi_1(S^1)$ and the fundamental theorem of algebra. Van Kampen's theorem. Fundamental group of a CW-complex. Covering spaces.

Homology. Simplicial homology, singular homology and their basic properties. Applications: Brouwer fixed point theorem, degree of a map from the sphere to itself, hairy ball theorem, Euler characteristic, Mayer-Vietoris sequence. Universal coefficient theorem and Künneth formula (statement).

Cohomology. The universal coefficient theorem (statement). Cup product. Applications: the Borsuk-Ulam and ham-sandwich theorems.

Reference:

1. A. Hatcher, *Algebraic Topology.*