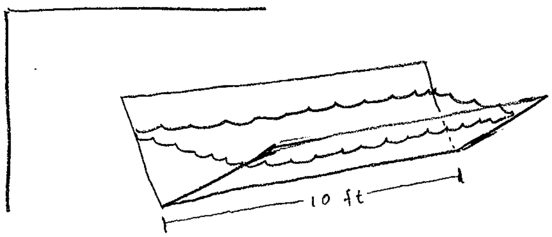
MATH 1A

Solutions

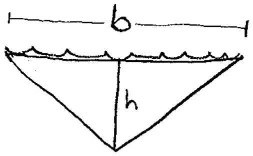
Quiz Four

(1) A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft/min, how fast is the water level rising when the water is 6 inches deep?



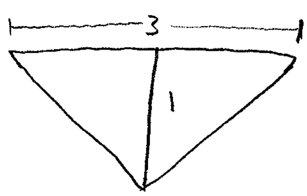
The volume of the water (shaped like a prism) is

$$V = \text{base area} \times \text{length}$$
$$= \frac{1}{2}(bh) \times 10$$
$$= 5bh.$$



But we need an equation with only V's and h's...

This triangle is similar to the big one:



so $\frac{b}{h} = \frac{3}{1}$, and $b = 3h$. Plug this into the volume formula:

$$V = 5(3h)\cdot h = 15h^2$$

and take the derivative:

$$\frac{dV}{dt} = 30h\frac{dh}{dt}.$$

Plug in what we know: $\left(\frac{dV}{dt} = 12 \text{ and } h = \frac{1}{2}\right)$ and solve:

$$\boxed{\frac{dh}{dt} = \frac{4}{5}\ \text{ft/min}}$$

(2) on back →

(2) If $g(x) + x\sin g(x) = x^2$, find $g'(0)$.

Use implicit differentiation:

$$g'(x) + \sin(g(x)) + x\cos(g(x))\,g'(x) = 2x.$$

Plug in $x=0$:

$$g'(0) + \sin(g(0)) + 0 = 0.$$

$$g'(0) = -\sin(g(0)).$$

Find $g(0)$ by plugging in $x=0$ in the original equation:

$$g(0) + 0 = 0, \quad \text{so} \quad g(0) = 0.$$

Hence $g'(0) = -\sin(0) = \boxed{0}$.