MATH 55 WORKSHEET 9

1 Express the gcd of 34 and 55 as a linear combination of 34 and 55.

Eucl. Algorithm:

$55 = 34\cdot 1 + 21$
$34 = 21\cdot 1 + 13$
$21 = 13\cdot 1 + 8$
$13 = 8\cdot 1 + 5$
$8 = 5\cdot 1 + 3$
$5 = 3\cdot 1 + 2$
$3 = 2\cdot 1 + \boxed{1}$
$2 = 1\cdot 1 + 0$

$\gcd(34,55) = 1.$

Then by very difficult and unenlightening substitution, we arrive at

$$\boxed{55\cdot 13 - 34\cdot 21 = 1}$$

The interesting thing to see is that everything in this problem is a Fibonacci number!

2 Solve the linear congruence $34x = 2 \pmod{55}$.

$34x \equiv 2 \pmod{55}$, so $\overline{34}\,34x \equiv 1 \equiv \overline{34}\cdot 2 \pmod{55}$.

By (1) we see that $55\cdot 13 - 34\cdot 21 = 1$, so
$-34\cdot 21 \equiv 1 \pmod{55}$. So $\overline{34} \equiv -21 \equiv 34 \pmod{55}$

And $X = 34\cdot 2 = 68 \equiv \boxed{13} \pmod{55}$

3 Find all solutions to: $X \equiv 1 \pmod 2$ $\quad X \equiv 2 \pmod 3$ $\quad X \equiv 3 \pmod 5$.

By CRT: $X \equiv \underbrace{1\cdot(3\cdot 5)\cdot\overline{(3\cdot 5)}} + \underbrace{2\cdot(2\cdot 5)\overline{(2\cdot 5)}} + \underbrace{3\cdot(2\cdot 3)\overline{(2\cdot 3)}}$
$= 1\cdot 15\cdot 1 + 2\cdot 10\cdot 1 + 3\cdot 6\cdot 1$
$= 53 \equiv \boxed{23} \pmod{2\cdot 3\cdot 5}$

4 Find all solutions, if any, to: $X \equiv 7 \pmod 9$, $X \equiv 4 \pmod{12}$, $X \equiv 16 \pmod{21}$.

We can't use CRT because 9, 12, and 21 aren't coprime.

But we can deduce from these three congruences that

$X \equiv 7 \equiv 1 \pmod 3$
$X \equiv 4 \equiv 0 \pmod 4$
$X \equiv 16 \equiv 2 \pmod 7$

Now use CRT on these new congruences:

$X \equiv 1\cdot(28)\overline{(28)} + 0\cdot(21)\overline{(21)} + 2\cdot(12)\overline{(12)}$
$= 1\cdot 28\cdot 1 + 0 + 2\cdot 12\cdot 3$
$= 100 \equiv \boxed{16} \pmod{84}$