Math 55 Worksheet 10

[1] Show that $n! = O(n^n)$.

$$n! = n\cdot(n-1)(n-2)\cdots 2\cdot 1 < n\cdot n\cdot n\cdots n\cdot n = n^n$$

$$C = 1 \qquad k = 1$$

[2] Show that n^n is not $O(n!)$

Suppose $n^n = O(n!)$. Then $n^n < Cn!$, so $\frac{n^n}{n!} < C$.

But $\frac{n^n}{n!} = \frac{n}{n}\cdot\frac{n}{n-1}\cdot\cdots\cdot\frac{n}{2}\cdot\frac{n}{1} > 1\cdot 1\cdot 1\cdot\cdots\cdot 1\cdot n = n$,

which is unbounded. Contradiction.

[3] Show $\log(x^2+1) = O(\log x)$.

$$\log(x^2+1) < \log(x^2+x^2) = \log(2x^2) = \log 2 + 2\log x = O(\log x).$$

[4] Find a big-O estimate for $f(x) = (x+1)\log(x^2+1) + 3x^2$.

$$f(x) = O\left[(x+1)\log x + 3x^2\right] \qquad (\text{by } \boxed{3})$$

$$= O\left[x\log x + x^2\right]$$

$$= O\left[x^2\right]$$