Math 55 Midterm 2 review problems:

1. How many ways are there to choose 6 items from 10 distinct items when
   (a) The items are ordered and repetition is not allowed?
   (b) The items are ordered and repetition is allowed?
   (c) The items are unordered and repetition is not allowed?
   (d) The items are unordered and repetition is allowed?

2. How many bit strings of length 10 either start with 000 or end with 1111?

3. How many positive integers less than 1000
   (a) have at least one digit equal to 9?
   (b) have no odd digits?
   (c) have two consecutive fives?
   (d) are palindromes?

4. Show that given any set of 10 positive integers not exceeding 50 there exist at least two different 5-element subsets of this set that have the same sum.

5. Show that in any set of $n+1$ positive integers not exceeding $2n$ there must be two that are relatively prime.

6. Find n if
   (a) $\binom{n}{2} = 45$ (b) $\binom{n}{5} = \binom{n}{2}$ (c) $\binom{n}{3} = P(n,2)$.

7. What is the probability that a hand of 13 cards contains no pairs?

8. What is the probability of dealing out all 52 cards to 4 people such that each person gets an ace?

9. Suppose p and q are distinct primes. What is the probability that a randomly chosen positive integer less than pq is divisible by either p or q?

10. What is the probability that a bit string of length n is a palindrome?

11. What is the probability that a random 5-letter word has its letters in alphabetical order?

12. What is the variance of the sum of a roll of a 6-sided die and an 8-sided die?

$\boxed{13}$ Find $\gcd(2n+1, 3n+2)$.

$\boxed{14}$ Show that $9 \mid n$ iff the sum of the digits of n is divisible by 9.

$\boxed{15}$ Show that $11 \mid n$ iff the <u>alternating sum</u> of the digits of n is divisible by 11.

$\boxed{16}$ Prove that if the sum of the divisors of n is $n+1$ then n is prime.

$\boxed{17}$ Explain why 123123, 492492, 641641, and any other 6-digit number of this form is divisible by 13.

$\boxed{18}$ Find the sum of all positive integers less than or equal to 100 that have an odd number of factors.

$\boxed{19}$ Find all solutions to the system of congruences

$$x \equiv 4 \pmod{6}$$
$$x \equiv 13 \pmod{15}$$

$\boxed{20}$ Show there are no solutions to the system

$$x \equiv 2 \pmod{6}$$
$$x \equiv 3 \pmod{9}.$$