Math 55 HW 9 Solutions

3.5 : 8
3.6 : 24
3.7 : 2a-c, 6, 12, 14, 18, 26, 28

$\boxed{3.5.8}$ Since $k \mid (n+1)!$ for every k from 2 to $n+1$, we see that $2 \mid (n+1)!+2$, $3 \mid (n+1)!+3$, and in general $k \mid (n+1)!+k$ for every k from 2 to $n+1$. Hence $\{(n+1)!+2, (n+1)!+3, \ldots, (n+1)!+n+1\}$ is a set of n consecutive composite integers.

$\boxed{3.6.24}$

(a) $\gcd(1,5) = 1$

(b) $\gcd(100, 101) = 1$

(c) $\gcd(123, 277) = 1$

(d) $\gcd(1529, 14039) = 139$

(e) $\gcd(1529, 14038) = 1$

(f) $\gcd(11111, 111111) = 1$.

$\boxed{3.7.2}$ (a) $1 = 9\cdot(5) + 11\cdot(-4)$ or $1 = 9(-6) + 11(5)$

(c)
① $78 = 35\cdot 2 + 8$
② $35 = 8\cdot 4 + 3$
③ $8 = 3\cdot 2 + 2$
④ $3 = 2\cdot 1 + 1$
$2 = 1\cdot 2 + 0$

$$
\begin{aligned}
1 &= 3 - 2 && (\text{by } ④)\\
&= 3 - (8 - 3\cdot 2) && (\text{by } ③)\\
&= 3\cdot 3 - 8\\
&= (35 - 8\cdot 4)\cdot 3 - 8 && (\text{by } ②)\\
&= 35\cdot(3) - 8(13)\\
&= 35\cdot(3) - (78 - 35\cdot 2)\cdot 13 && (\text{by } ①)\\
&= \boxed{35\cdot(29) - 78(13)}
\end{aligned}
$$

$\boxed{6}$ $\overline{2} \equiv 9$ because $2\cdot 9 = 18 \equiv 1 \pmod{17}$.

$\boxed{12}$ Since $\overline{2} \equiv 9$, multiply both sides by 9:

$$2x \equiv 7 \pmod{17} \longrightarrow 9\cdot 2x \equiv 9\cdot 7 \pmod{17} \longrightarrow x \equiv 12 \pmod{17}$$

$\boxed{14}$ (a)
$2\cdot 6 = 12 \equiv 1 \pmod{11}$
$3\cdot 4 = 12 \equiv 1 \pmod{11}$
$5\cdot 9 = 45 \equiv 1 \pmod{11}$
$7\cdot 8 = 56 \equiv 1 \pmod{11}$

(b)
$$
\begin{aligned}
10! &= 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\\
&= 10\cdot(2\cdot 6)\cdot(3\cdot 4)\cdot(5\cdot 9)\cdot(7\cdot 8)\cdot 1\\
&\equiv 10\cdot 1\cdot 1\cdot 1\cdot 1\cdot 1\\
&= 10 \equiv -1 \pmod{11}.
\end{aligned}
$$

$\boxed{18}$ $X \equiv 2\cdot(4\cdot5)\cdot\overline{(4\cdot5)} + 1\cdot(3\cdot5)\overline{(3\cdot5)} + 3\cdot(3\cdot4)\overline{(3\cdot4)}$

$= 2\cdot 20\cdot 2 + 1\cdot 15\cdot 3 + 3\cdot 12\cdot 3$

$= 80 + 45 + 108$

$= 233 \equiv \boxed{53} \pmod{60}$

So all solutions are $\boxed{X = 53 + 60n}$

$\boxed{26}$ $X \equiv 0 \pmod 5$

$X \equiv 1 \pmod 3$

$X \equiv 0\cdot 3\cdot\bar{3} + 1\cdot 5\cdot\bar{5}$

$= 0 + 1\cdot 5\cdot 2 = 10$

so $\boxed{X = 10 + 15n}$

$\boxed{28}$ (a) FLT: $3^4 \equiv 1 \pmod 5$, $3^6 \equiv 1 \pmod 7$, $3^{10} \equiv 1 \pmod{11}$.

so $3^{302} = (3^4)^{75}3^2 \equiv 3^2 \equiv 4 \pmod 5$.

$3^{302} = (3^6)^{50}3^2 \equiv 3^2 \equiv 2 \pmod 7$.

$3^{302} = (3^{10})^{30}3^2 \equiv 3^2 \equiv -2 \pmod{11}$.

(b) $X = 3^{302}$.

$X \equiv 4 \pmod 5$

$X \equiv 2 \pmod 7$

$X \equiv -2 \pmod{11}$

$\xrightarrow{\text{CRT}}$

$X \equiv 4(7\cdot11)\overline{(7\cdot11)} + 2(5\cdot11)\overline{(5\cdot11)} + {}^-2(5\cdot7)\overline{(5\cdot7)}$

$= 4\cdot 77\cdot 3 + 2\cdot 55\cdot 6 + -2\cdot(35)\cdot 6$

$= 1164 \equiv \boxed{9} \pmod{385}$