# Math 55 HW 15 solutions

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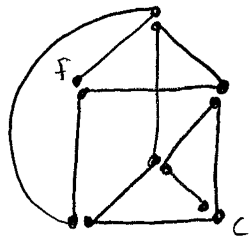
|---|
| 9.5: 4, 26, 62 |
| 9.7: 3, 25 |
| 9.8: 2, 4, 16 |

**9.5.4** $\deg(f) = \deg(c) = 3$, so path, not circuit:




**26** (a) In $K_n$, $\deg(v) = n-1$ $\forall v \in V$. So $K_n$ has an Euler circuit when $n$ is odd.

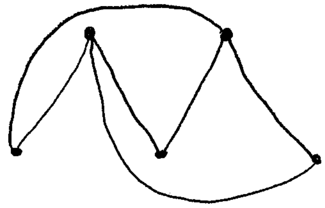
(b) In $C_n$, $\deg(v) = 2$ $\forall v \in V$. So $C_n$ always has an Euler circuit.

(c) In $W_n$, $\deg(v) = 3$ $\forall v \in V$, so $W_n$ never has an Euler circuit.
(except the hub, which has degree $n$. But that won't affect the result.)

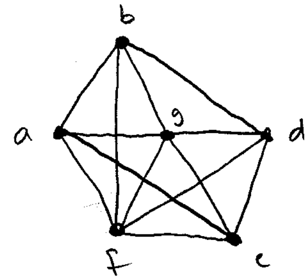
(d) In $Q_n$, every vertex is represented by a bitstring of length $n$. Any such bit string differs from $n$ other bit strings in exactly one bit. Hence $\deg(v) = n$ for all vertices in $Q_n$. So $Q_n$ has an Euler circuit iff $n$ is even.

**62** Color the vertices according to the colors of the corresponding squares on the chess board. You'll notice that any legal move will move the knight to a square of opposite color. This coloring forms our bipartition.

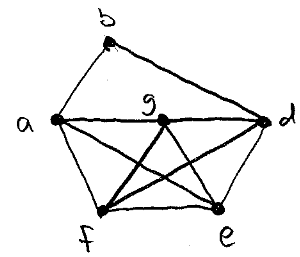
**9.7.3**



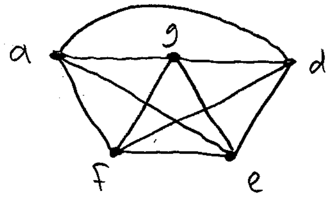
25 Look at graph with c removed:



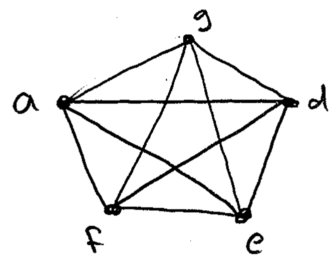

Now remove edges bf and bg:




Finally, remove vertex b by fusing ab and bd into one edge: ad:



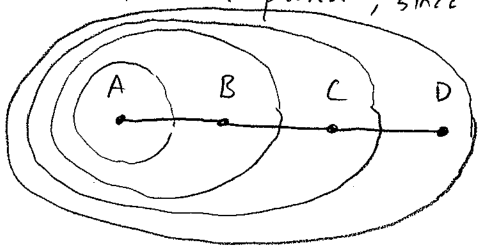

Moving vertex g up a few inches reveals ...




$= K_5$ !

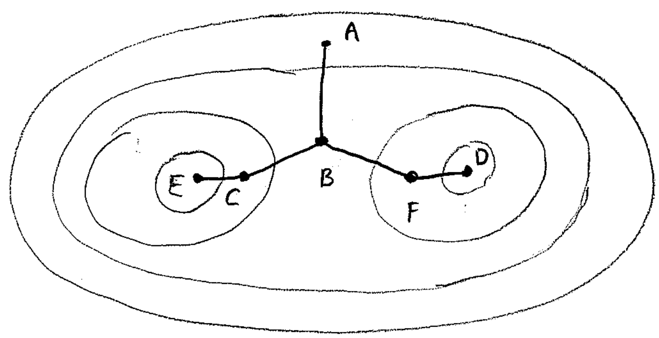
So it's not planar, since it contains a subgraph homeomorphic to $K_5$.

9.8.2




2 colors needed.

4




2 colors needed.

16 The only way to color a circuit with 2 colors is by alternating the colors. This can't happen in an odd cycle:

e.g.: