

MATH 54

sample midterm II

(1) If possible, diagonalize $A = \begin{bmatrix} 7 & 4 & 16 \\ 2 & 5 & 8 \\ -2 & -2 & -5 \end{bmatrix}$

(2) Let $V =$ the subspace of $C[-1, 1]$ spanned by $1, t,$ and t^2 . Using the inner product $\langle u, v \rangle = \int_{-1}^1 uv$ find the distance from t^3 to V .

(3) Is there a unique best-fit parabola $y = ax^2 + bx + c$ for the data set: $(1, 0), (1, 2), (3, 1), (2, 2)$? Explain.

(4) Write out the spectral decomposition of

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

(5) Let $M = \begin{bmatrix} \boxed{A} & \boxed{B} \\ & \boxed{C} & \boxed{D} \\ & & & \boxed{E} \end{bmatrix}$ be a block upper triangular matrix with $A, C,$ and E invertible, of

dimensions $A: 5 \times 5, C: 2 \times 2, E: 2 \times 2$.

Find a formula for the block matrix M^{-1} in terms of $A, B, C, D,$ and E .

(6) Try the suggested supplementary exercises. At least one of the theoretical ones will show up on the exam!

(7) Prove that if $\{u, v\}$ is an orthonormal set, then $\|u - v\| = \sqrt{2}$.