MATH 54: PRACTICE FINAL PROBLEMS (I)

(1) Find a general solution to

(a) $y'' + 16y = te^t$

(b) $y^{(4)} = 120t$

(c) $y'' - 8y' - 33y = 546 \sin t$

(d) $y'' + 16y = \tan 4t$.

(2) Find the solution to the IVP: $y'' + y = \sec\theta$, $y(0) = 1$, $y'(0) = 2$.

(3) Let n be a positive integer. Show $\{1, x, x^2, \ldots, x^n\}$ is a linearly independent set. (Use the fact that a degree-n polynomial has at most n roots unless it is the zero polynomial.)

(4) Find a general solution to

(a) $x' = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix} x$.

(b) $x' = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} x$.

(c) $x' = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} x + \begin{bmatrix} 2e^t \\ 4e^t \end{bmatrix}$

(5) Find the solution to the IVP: $x' = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} x + \begin{bmatrix} e^t \\ -e^t \end{bmatrix}$, $x(0) = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$.

(6) Describe the vector field of solutions of $y'' - 4y' + 5y = 0$.

(7) Review old homework problems and quizzes from the whole summer! A few familiar old problems will appear on the final.