MATH 54 wishes you a Merry Quiz-six

Solutions!

1 Let $W = \text{Span}\{x_1, x_2\}$, where $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1/3 \\ 1/3 \\ -2/3 \end{bmatrix}$.

Construct an orthonormal basis for W.

Check if x_1 and x_2 are orthogonal:

$$x_1 \cdot x_2 = \tfrac{1}{3} + \tfrac{1}{3} - \tfrac{2}{3} = 0.$$ yes, they are!

Now normalize:

$$\frac{x_1}{\|x_1\|} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \qquad \frac{x_2}{\|x_2\|} = \frac{1}{\sqrt{6/9}}\begin{bmatrix} 1/3 \\ 1/3 \\ -2/3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{bmatrix}.$$

2 True or False:

(a) If A is symmetric, so is A^2.

$(A^2)^T = (A \cdot A)^T = A^T A^T = A \cdot A = A^2$. True.

(b) If A is orthogonally diagonalizable, so is A^2.

ortho. diag = symmetric. True.

— or —

$A^2 = A \cdot A = SDS^TSDS^T = SD^2S^T$. True.