

MATH 54 ~~talk~~ 5 Solutions

1] Let $A, B,$ and C be matrices with characteristic polynomials

$$\chi_A(\lambda) = \lambda(\lambda-2)^2(\lambda+1), \quad \chi_B(\lambda) = \lambda^4 - 16,$$

and $\chi_C(\lambda) = \lambda(\lambda-1)(\lambda+1)$. Circle all that apply:

(a) Which of $A, B,$ and C are not invertible? A B C

(b) Which of $A, B,$ and C must be diagonalizable over \mathbb{R} ?

A B C

(c) What are the dimensions of A ?

3x3 4x4 cannot be determined.

Explanation: (a) matrices A and C have 0 as an e -value.
 (b) only C has real distinct e -values
 (c) degree of $\chi_A(\lambda)$ is 4.

2] What is the distance between the point $(4, 3)$ and the line $y = 2x$?

Let $v = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, and $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. (u is basis for the line).

Then $\hat{v} = \text{proj}_u(v) = \frac{v \cdot u}{u \cdot u} u = \frac{10}{5} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

So the distance we want is $\|v - \hat{v}\| = \left\| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\| = \sqrt{5}$.

