

Math 54 Quiz three name: Solutions!

Let $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix}$.

① Find a basis for $\text{Col } A$.

First row-reduce:

$A \xrightarrow[\substack{R2 := R2 - 2R1 \\ R3 := R3 - R1}]{R2 := R2 - 2R1} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Pivots are in

columns 1 and 2, so a basis for $\text{Col } A$ is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right\}$.

② Find a basis for $\text{Nul } A$.

Finish row reducing:

$\xrightarrow{-\frac{1}{3}R2} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R1 := R1 - 2R2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Now write solution of $AX=0$ parametrically:

$\left\{ \begin{bmatrix} -t \\ -u \\ t \\ u \end{bmatrix} \right\} = \left\{ t \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$. Basis = $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

③ Find the determinant of the matrix formed by removing the last column of A .

$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$
 $= -3 - 0 + 3 = 0$.