

Math 54 HW 1 selected solutions

1.1: 8, 14, 16, 20, 30

$$(16) \begin{bmatrix} 1 & 0 & 0 & -1 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix} \xrightarrow[\frac{1}{2}R_2]{R_4 := R_4 + 2R_1} \begin{bmatrix} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -1 & -1 \end{bmatrix} \xrightarrow{R_4 := R_4 - 3R_2} \begin{bmatrix} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} \xrightarrow{R_4 := R_4 + R_3} \begin{bmatrix} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \quad \boxed{\text{consistent}}$$

$$(20) \begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix} \xrightarrow{R_2 := R_2 + 2R_1} \begin{bmatrix} 1 & h & -3 \\ 0 & 4+2h & 0 \end{bmatrix}$$

This will be consistent for any h , because even if $h = -2$, there will not be a pivot in the last column.

(30) $R_2 := -\frac{1}{2}R_2$ and $R_2 := -2R_2$.

1.2: 1, 2, 4, 6, 12, 16, 20, 28

- (2) a. reduced echelon
- b. echelon
- c. neither
- d. echelon

$$(4) \begin{bmatrix} \textcircled{1} & 3 & 5 & 7 \\ 3 & \textcircled{5} & 7 & 9 \\ 5 & 7 & 9 & \textcircled{1} \end{bmatrix} \xrightarrow[\begin{matrix} R_2 := R_2 - R_1 \\ R_3 := R_3 - R_2 \end{matrix}]{\begin{matrix} R_2 := R_2 - 2R_1 \\ R_3 := R_3 - R_2 \end{matrix}} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & -8 \end{bmatrix} \xrightarrow[\begin{matrix} \frac{1}{4}R_2 \\ \frac{1}{10}R_3 \end{matrix}]{\begin{matrix} R_2 := R_2 - 2R_1 \\ R_3 := R_3 - R_2 \end{matrix}} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[\begin{matrix} R_1 := R_1 - 7R_3 \\ R_2 := R_2 - 3R_3 \end{matrix}]{R_1 := R_1 - 3R_2} \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 := R_1 - 3R_2} \begin{bmatrix} \textcircled{1} & 0 & -1 & 0 \\ 0 & \textcircled{1} & 2 & 0 \\ 0 & 0 & 0 & \textcircled{1} \end{bmatrix}$$

(6) $\begin{bmatrix} \blacksquare & \times \\ 0 & \blacksquare \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} \blacksquare & \times \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$(17) \begin{bmatrix} 1 & -7 & 0 & 6 & | & 5 \\ 0 & 0 & 1 & -2 & | & -3 \\ -1 & 7 & -4 & 2 & | & 7 \end{bmatrix} \xrightarrow{R_3 := R_3 + R_1} \begin{bmatrix} 1 & -7 & 0 & 6 & | & 5 \\ 0 & 0 & 1 & -2 & | & -3 \\ 0 & 0 & -4 & 8 & | & 12 \end{bmatrix} \xrightarrow{R_3 := R_3 + 4R_2} \begin{bmatrix} 1 & -7 & 0 & 6 & | & 5 \\ 0 & 0 & 1 & -2 & | & -3 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$x_1 = 5 + 7t - 6u$$

$$x_2 = t$$

$$x_3 = -3 + 2u$$

$$x_4 = u$$

$$\text{solution set} = \left\{ \begin{bmatrix} 5 \\ 0 \\ -3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} -6 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

(16) (a) consistent + unique

(b) consistent + not unique

$$(20) \begin{bmatrix} 1 & 3 & | & 2 \\ 3 & h & | & k \end{bmatrix} \xrightarrow{R_2: R_2 - 3R_1} \begin{bmatrix} 1 & 3 & | & 2 \\ 0 & h-9 & | & k-6 \end{bmatrix}$$

(a) no solution if $h=9$ and $k \neq 6$

(b) unique solution if $h \neq 9$

(c) ∞ solns if $h=9$ and $k=6$.

(28) That every column except the augmented column is a pivot column.