HW 11, Nov. 6th

4.3 10. $g(x) = (e^{-2x} - 2x)^3$

$$g'(x) = 3(e^{-2x} - 2x)^2((-2)e^{-2x} - 2)$$

$$\boxed{= -6(e^{-2x} - 2x)^2(e^{-2x} + 1)}$$

13. $y = e^{x^2 - 5x + 4}$

$$\frac{dy}{dx} = \boxed{e^{x^2 - 5x + 4}(2x - 5)}$$

14. $g(t) = t^2 e^{1/t}$

$$g'(t) = 2te^{1/t} + t^2\left(-\frac{1}{t^2}\right)(e^{1/t}) \boxed{= (2t - 1)e^{1/t}}$$

20. $f(x) = \sqrt{e^{x/2} + 1}$

$$f'(x) = \frac{1}{2\sqrt{e^{x/2}+1}} \cdot \frac{e^{\frac{x}{2}}}{2} = \boxed{\frac{e^{x/2}}{4\sqrt{e^{x/2} + 1}}}$$

22. $f(x) = e^{e^x}$

$$f'(x) = e^x e^{e^x} = \boxed{e^{x + e^x}}$$

23. $f(x) = \frac{e^{4x}}{4 + x}$

$$f'(x) = \frac{4e^{4x}(4+x) - e^{4x}}{(4+x)^2} = \frac{(4x + 15)e^{4x}}{(4+x)^2}$$

26. $f(x) = \dfrac{4x^2}{x^2+e^{2x}}$

$$f'(x) = \frac{8x(x^2+e^{2x}) - 4x^2(2x+2e^{2x})}{(x^2+e^{2x})^2} = \frac{4x^3+8(x-x^2)e^{2x}}{(x^2+e^{2x})^2}$$

31. $f(x) = (5x-2)e^{1-2x}$

$$f'(x) = 5e^{1-2x} + (5x-2)(-2)e^{1-2x}$$

$$= (-10x+9)e^{1-2x} = 0 \quad \text{when} \quad x = \frac{9}{10} \quad (e^{1-2x} \text{ is never } 0)$$

$$f''(x) = -10e^{1-2x} + (-10x+9)(-2)e^{1-2x}$$

$$= (20x-28)e^{1-2x} < 0 \quad \text{when} \quad x = \frac{9}{10} \quad \text{so } \boxed{\text{local max}}$$

39. $y = e^{-2e^{-\frac{x}{100}}}$

$$\frac{dy}{dx} = e^{-2e^{-\frac{x}{100}}}\left(\frac{1}{50}\right)e^{-\frac{x}{100}} = \boxed{\frac{e^{\left(-2e^{-\frac{x}{100}}-\frac{x}{100}\right)}}{50}}$$

48. $$\lim_{h\to 0}\frac{e^{2h}-1}{h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \quad \text{where} \quad f(x)=e^{2x}, \; a=0$$

$$= \left.\frac{d}{dx}\right|_{x=0} e^{2x} = \left.2e^{2x}\right|_{x=0} \boxed{=2}$$

4.4.18 $e^{\ln 3 - 2\ln x} = e^{\ln \frac{3}{x^2}} = \boxed{\frac{3}{x^2}}$

21. $\ln(4-x) = \frac{1}{2}$

$e^{\ln(4-x)} = e^{\frac{1}{2}}$

$4 - x = \sqrt{e}$

$\boxed{x = 4 - \sqrt{e}}$

32. $e^{\sqrt{x}} = \sqrt{e^x}$

$e^{2\sqrt{x}} = e^x$

$\ln e^{2\sqrt{x}} = \ln e^x \Rightarrow 2\sqrt{x} = x \Rightarrow \sqrt{x} = 2$ or $x = 0$

$\boxed{x = 4, 0}$

38. $(e^x)^2 e^{2-3x} = 4 \Rightarrow e^{2-3x+2x} = 4$

$e^{2-x} = 4 \Rightarrow 2 - x = \ln 4 \Rightarrow \boxed{x = 2 - \ln 4}$

39. $f(x) = -5x + e^x$

$f'(x) = -5 + e^x = 0 \Rightarrow e^x = 5 \Rightarrow x = \ln 5$ critical pt.

$f''(x) = e^x > 0$ at $\ln 5$, so it's a min

$f(\ln 5) = -5\ln 5 + 5$

$\boxed{(\ln 5, -5\ln 5 + 5)}$

40. $f(x) = -1 + (x-1)^2 e^x$

$$f'(x) = 2(x-1)e^x + (x-1)^2 e^x$$

$$= (x^2 - 2x + 1 + 2x - 2)e^x = (x+1)(x-1)e^x = 0 \text{ when } x = -1, 1$$

so from picture, max at $x=-1$, min at $x=1$

$$f(-1) = -1 + 4e^{-1} = \frac{4}{e} - 1 \qquad \boxed{\left(-1, \frac{4}{e} - 1\right)} \text{ max}$$

$$f(1) = -1 + 0 = -1 \qquad \boxed{(1, -1)} \text{ min}$$

43. $f(x) = e^{-x} + 3x$, $\quad f'(x) = 3 - e^{-x} = 0$

when $-x = \ln 3 \Rightarrow x = -\ln 3 = \ln \frac{1}{3}$

$f''(x) = e^{-x} > 0$ for all x, so it's a local min.

$$f(-\ln 3) = 3 - 3\ln 3 \qquad \boxed{(-\ln 3, 3 - 3\ln 3)}$$

5. $y = \ln(e^x + e^{-x})$

$$\boxed{y' = \frac{e^x - e^{-x}}{e^x + e^{-x}}}$$

12. $y = \dfrac{\ln x}{\ln 2x}$

$$\frac{dy}{dx} = \frac{\frac{1}{x}(\ln 2x) - \frac{1}{2x}(\ln x)}{(\ln 2x)^2}$$

$$= \frac{2\ln 2x - \ln x}{2x(\ln 2x)^2}$$

15. $y = (x^2+1)\ln(x^2+1)$

$$\frac{dy}{dx} = 2x\ln(x^2+1) + (x^2+1)\frac{2x}{x^2+1} = 2x\left(\ln(x^2+1)+1\right)$$

18. $y = \left[\ln(e^{2x}+1)\right]^2$

$$\boxed{\frac{dy}{dx} = 2\left[\ln(e^{2x}+1)\right]\frac{2e^{2x}}{e^{2x}+1}}$$

20. $y = \sqrt{\ln 2x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\ln 2x}} \cdot \frac{1}{2x} = \boxed{\frac{1}{4x\ln\sqrt{2x}}}$$

23. $$\frac{d^2}{dt^2}(\ln t)^3 = \frac{d}{dt}\left(\frac{3(\ln t)^2}{t}\right) = \frac{6\ln t}{t^2} - \frac{3(\ln t)^2}{t^2}$$ ← product rule on $3(\ln t)^2 \cdot \frac{1}{t}$

$$= \boxed{\frac{6\ln t - 3(\ln t)^2}{t^2}}$$

25. $y = \ln(x^2+e)$ @ $x=0$

$y(0) = \ln(0+e) = 1$ point: $(0,1)$

$y'(0) = \left.\frac{2x}{x^2+e}\right|_{x=0} = 0$ slope $= 0$

$$y-1 = 0(x-0) \Rightarrow \boxed{y=1}$$

27. $f(x) = \dfrac{(\ln x)}{\sqrt{x}}$

$$f'(x) = \frac{\frac{\sqrt{x}}{x} - \frac{\ln x}{2\sqrt{x}}\left(\frac{2x}{2x}\right)}{x} = \frac{2\sqrt{x} - \sqrt{x}\ln x}{2x^2} = 0$$

not in domain of f

$$\sqrt{x}\,(2 - \ln x) = 0 \Rightarrow x = \cancel{0} \text{ or } e^2, \quad \underline{f(e^2) = \frac{2}{e}}$$

by picture, this is a max, so $\boxed{\left(e^2, \frac{2}{e}\right)}$

32. $C(x) = \dfrac{1000 \ln x}{\sqrt{100 - 3x}}$ find $C'(25)$

$$C'(25) = \left.\frac{\frac{1000}{x}\sqrt{100-3x} - \frac{1000 \ln x}{2\sqrt{100-3x}}(-3)}{100-3x}\right|_{x=25} \qquad (\ln 25 = 2\ln 5)$$

$$= \frac{40\sqrt{25} + \frac{3000 \ln 5}{\sqrt{25}}}{25}$$

$$= \frac{200 + 600 \ln 5}{25} = \boxed{8 + 24 \ln 5}$$