Name Di **Math 221 Quiz 3** Section Mooncake Score ____/10

Instructions. Inform your TA if you find any error in the sdutions. ☺
Please make sure to SHOW YOUR WORK, and do not skip steps.

1. *(3 points)* $\lim_{x\to-\infty} \frac{9x^2+x^{-1}+2016}{29x-x^2+221}$

$$= \lim_{x\to-\infty} \frac{9x^2 - x^{-1} + 2016}{29x - x^2 + 221} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{9 - \frac{1}{x^3} + \frac{2016}{x^2}}{\frac{29}{x} - 1 + \frac{221}{x^2}}$$

$$= -9$$

2. *(4 points)* $\lim_{x\to+\infty} \left(\sqrt{x^2+2x} - x\right)$ $\frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x}$

$$= \lim_{x\to+\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x}$$

$$= \lim_{x\to+\infty} \frac{2x}{\sqrt{x^2+2x}+x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to+\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1}$$

$$= \frac{2}{1+1} = 1$$

3. *(3 points)* $\lim_{x\to0} x^{100}\cos\left(\frac{1}{\sqrt{|x|}}\right)$. (You need to justify your answer.)

$$-1 \le \cos\left(\frac{1}{\sqrt{|x|}}\right) \le 1$$

$$-x^{100} \le x^{100}\cos\left(\frac{1}{\sqrt{|x|}}\right) \le x^{100}$$

$\searrow 0$ as $x\to0$ $\quad \swarrow 0$

$$\Rightarrow \lim_{x\to0} x^{100}\cos\left(\frac{1}{\sqrt{|x|}}\right) = 0 \quad \text{by sanduich thm.}$$

P.S. In your discussion, the question might be slightly different. For example, $x^{100}\cos\left(\frac{1}{\sqrt{x^2}}\right)$, $x^{100}\cos\left(\frac{1}{\sqrt{x^2+x^4}}\right)$, ...
But the idea is the same!