This quiz came from the Early Alert System, and is meant to be hard.
Do not freak out too much, and feel free to talk to your TA.
It's always good to be aware of some weak links at an early stage! :)

EARLY ALERT SYSTEM — FALL 2016
MATH 221

Name: ____________________________________

Give an answer. No calculators are allowed.

1. Find the slope of the line going through points $(-1, \frac{2}{3})$ and $(\frac{-2}{3}, -2)$. What is the equation of this line?

Pay attention to computation!

$$\text{Slope} = \frac{\frac{2}{3} - (-2)}{-1 - (-\frac{2}{3})} = \frac{\frac{2}{3}+2}{-1+\frac{2}{3}} = \frac{\frac{8}{3}}{-\frac{1}{3}} = -8$$

$$\text{line: } y - \frac{2}{3} = -8(x+1)$$

Or

$$y = mx + b.$$

$$m = \frac{\frac{2}{3} - (-2)}{-1 - (-\frac{2}{3})} = -8$$

Plug in $(-1, \frac{2}{3})$

$$\frac{2}{3} = -8 \times (-1) + b$$

$$\Rightarrow b = \frac{2}{3} - 8 = -\frac{22}{3} \qquad \Rightarrow y = -8x - \frac{22}{3}$$

2. Rationalize the denominator of $\frac{h}{\sqrt{x+h} - \sqrt{x}}$.

$$= \frac{h(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}$$

$\rightarrow$ here use $(a-b)(a+b) = a^2 - b^2$

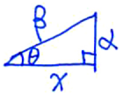
So $(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x}) = (\sqrt{x+h})^2 - (\sqrt{x})^2$

$$= \frac{h(\sqrt{x+h} + \sqrt{x})}{x+h-x}$$

$$= \sqrt{x+h} + \sqrt{x}$$

3. Calculate the $\tan(\theta)$ if $\sin(\theta) = \dfrac{\alpha}{\beta}$ and $0 < \theta < \dfrac{\pi}{2}$. (hint: All roads lead to Rome. One is to try drawing a straight triangle!)

$\boxed{\text{Way 1}}$ Draw a picture



$$\alpha^2 + x^2 = \beta^2$$

$$\Rightarrow x = \sqrt{\beta^2 - \alpha^2}$$

$$\tan\theta = \frac{\alpha}{x} = \frac{\alpha}{\sqrt{\beta^2 - \alpha^2}}$$

$\boxed{\text{Way 2}}$ Do it directly by $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Since $\sin^2\theta + \cos^2\theta = 1$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{\alpha^2}{\beta^2}} = \sqrt{\frac{\beta^2 - \alpha^2}{\beta^2}} = \frac{\sqrt{\beta^2 - \alpha^2}}{\beta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\alpha}{\beta}}{\frac{\sqrt{\beta^2 - \alpha^2}}{\beta}} = \frac{\alpha}{\sqrt{\beta^2 - \alpha^2}}$$

4. Find all real solutions of

$$\sqrt{x^2 + 9} = 4.$$

$$x^2 + 9 = 4^2$$

$$x^2 = 16 - 9 = 7$$

$$x = \pm\sqrt{7}$$

Don't forget "$\pm$"!

$(-\sqrt{7})^2$ is also 7!

5. Solve for x:

$$|2x - 1| \le 4$$

$$-4 \le 2x - 1 \le 4$$

$$-3 \le 2x \le 5$$

$$-\frac{3}{2} \le x \le \frac{5}{2}$$