Webwork due 10/01 Hints (by Di) for some sample questions

#1

Sample Q: $f(x) = x - 10\ln(3x)$ $f'(x) = ?$

Ans: $f'(x) = 1 - 10\,[\ln(3x)]'$ ⇐ chain rule! use Formula $\frac{d}{du}[\ln u] = \frac{1}{u}$

$= 1 - 10\,\frac{1}{3x} \cdot (3x)'$

outer derivative, inner derivative

$= 1 - 10 \cdot \frac{1}{3x} \cdot 3$

$= 1 - \frac{10}{x}$

#2 Sample Q:

$f(x) = \ln(9x+5) + 2017$ $f'(x) = ?$ Domain $= ?$

chain rule

Ans: $f'(x) = \frac{1}{9x+5} \cdot 9$

outer' inner'

Domain: $9x+5 > 0$

$9x > -5$

$x > -\frac{5}{9}$

$\ln x$ → $x > 0$ is the domain

#3 Sample Q: (Just some hints)

$(x^2+y^2)^2 = 15xy^2$ $y' = ?$

Hints: Differentiate the whole equation

$\frac{d}{dx}[(x^2+y^2)^2] = 2(x^2+y^2) \cdot \frac{d}{dx}[x^2+y^2]$

$= 2(x^2+y^2) \cdot (2x + 2y\frac{dy}{dx})$

Product rule!!!

$\frac{d}{dx}[15xy^2] = 15\frac{d}{dx}[xy^2]$

$= 15\left[\frac{d}{dx}x \cdot y^2 + x\frac{d}{dx}[y^2]\right]$

$= 15\left[y^2 + x \cdot 2y\frac{dy}{dx}\right]$

So

$$2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 15\left(y^2+2xy\frac{dy}{dx}\right)$$ want to solve for $\frac{dy}{dx}$

Multiply out

$$4x(x^2+y^2)+4y(x^2+y^2)\frac{dy}{dx} = 15y^2+30xy\frac{dy}{dx}$$

$$4y(x^2+y^2)\frac{dy}{dx} - 30xy\frac{dy}{dx} = 15y^2 - 4x(x^2+y^2)$$

$$\left(4y(x^2+y^2)-30xy\right)\frac{dy}{dx} = 15y^2-4x(x^2+y^2)$$

Finally !!! $\Rightarrow$ $$\frac{dy}{dx} = \frac{15y^2-4x(x^2+y^2)}{4y(x^2+y^2)-30xy}$$

#4 Sample Q:

Find slope of tangent line to $\sqrt{5x+y}+\sqrt{5xy} = 15$ at pt $(4,5)$

Ans: Re Write $(5x+y)^{\frac{1}{2}} + (5xy)^{\frac{1}{2}} = 15$

$$\frac{d}{dx}\left[(5x+y)^{\frac{1}{2}}\right] = \frac{1}{2}(5x+y)^{-\frac{1}{2}}\cdot\frac{d}{dx}[5x+y]$$

$$= \frac{1}{2}(5x+y)^{-\frac{1}{2}}\cdot\left(5+\frac{dy}{dx}\right)$$

$$\frac{d}{dx}\left[(5xy)^{\frac{1}{2}}\right] = \frac{1}{2}(5xy)^{-\frac{1}{2}}\cdot\frac{d}{dx}[5xy]$$ ↙ pull out constant.

$$= \frac{1}{2}(5xy)^{-\frac{1}{2}}\cdot 5\frac{d}{dx}[xy]$$ product rule.

$$= \frac{1}{2}(5xy)^{-\frac{1}{2}}\cdot 5\left[y+x\frac{dy}{dx}\right]$$

$\Rightarrow$ $$\frac{1}{2}(5x+y)^{-\frac{1}{2}}\left(5+\frac{dy}{dx}\right)+\frac{1}{2}(5xy)^{-\frac{1}{2}}5\left(y+x\frac{dy}{dx}\right)=0$$

Plug in $(x,y)=(4,5)$

$$\frac{1}{2}\cdot 25^{-\frac{1}{2}}\left(5+\frac{dy}{dx}\right)+\frac{1}{2}100^{-\frac{1}{2}}\cdot 5\left(5+4\frac{dy}{dx}\right)=0$$

$25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$ , $100^{-\frac{1}{2}} = \frac{1}{\sqrt{100}} = \frac{1}{10}$

$\Rightarrow \frac{1}{10}\left(5+\frac{dy}{dx}\right) + \frac{1}{4}\left(5+4\frac{dy}{dx}\right) = 0$

$\frac{1}{2} + \frac{1}{10}\frac{dy}{dx} + \frac{5}{4} + \frac{dy}{dx} = 0$

$\frac{11}{10}\frac{dy}{dx} = -\frac{5}{4} - \frac{1}{2} = -\frac{5}{4} - \frac{2}{4} = -\frac{7}{4}$

Finally $\frac{dy}{dx} = -\frac{7}{4} \cdot \frac{10}{11}$

$= \frac{-70}{44}$