David Hill
Office: Evans Hall, Room 785.
Office Hours: TBA
email: dhill1@math.berkeley.edu
phone: 642-6923

1. Math 257: Group Theory

1.1. General Information. In this course, we will study the representation theory of affine Hecke algebras of type A. This algebra has a family of remarkable finite dimensional quotients, including the group algebra of the symmetric group. We will approach this subject from various points of view. We will review classical Schur-Weyl duality and its affinization. Next, we will go on to investigate how this representation theory can be understood by studying highest weight representations of affine Kac-Moody algebras of type A. Time permitting, we will also explore connections to a new diagram calculus introduced by Khovanov and Lauda.

1.2. Homework. There will be occasional homework assignments, mostly at the beginning of the course. The idea is to acquaint you with the basic technical knowledge you will need.

As the course progresses, connections to other areas of mathematics will appear. You will be encouraged to pursue at least one such avenue and write a short report.

1.3. Important Dates.

- Administrative Holiday: 2/16
- Spring Break: 3/23-3/27
- Last day of instruction: 5/11

2. (Rough) Schedule

- **Weeks 1-3: Introductory Topics.** Background from representation theory of finite groups, Maschke's theorem, classical approach to representation theory of symmetric groups.
- **Weeks 4-7: (Degenerate) Affine Hecke algebras.** affine Hecke algebras of type A, weights and integral modules, the "Mackey theorem", formal characters and central characters, intertwiners, segment representations and standard modules, unique simple quotients.
- **Weeks 8-9: Affine Schur-Weyl Duality** Classical Schur-Weyl Duality, the Arakawa-Suzuki functor.
- **Weeks 10-12: Categorification** Representation theory of symmetric groups revisited, covering modules, crystal operators, crystals.
- **Weeks 13-14: Quantum Shuffle algebras** Classification of simple modules for affine Hecke algebras.
- **Week 15: Blocks of Cyclotomic Hecke algebras**

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