REPRESENTATION THEORY OF SYMMETRIC GROUPS AND HECKE ALGEBRAS

DAVID HILL

1. Segment Representations for Quiver Hecke Algebras of Classical Lie Algebras

1.1. **Good Lyndon Words.** Using the labeling of roots in [**?**, Section 8] and the costandard lexicographic ordering on Lyndon words, prove the following:

Proposition 1.1.1. *Show that the Good Lyndon words for $\mathfrak{g}$ of type A_r are*

$$\{[i,\dots,j]|1 \le i \le j \le r\}.$$

Proposition 1.1.2. *Show that the good Lyndon words for $\mathfrak{g}$ of type B_r are*

$$\{[i,\dots,j]|1 \le i \le j \le r\} \cup \{[j, j-1,\dots,1,1,\dots,k-1,k]|1 \le j < k \le r\}.$$

Proposition 1.1.3. *Show that the good Lyndon words for $\mathfrak{g}$ of type C_r are*

$$\{[i,\dots,j]|1 \le i \le j \le r\} \cup \{[j,\dots,2,1,2,\dots,k]|2 \le j < k \le r\} \cup \{[1,\dots,j,2\dots,j]|2 \le j \le r\}.$$

Proposition 1.1.4. *Show that the good Lyndon words for $\mathfrak{g}$ of type D_r are*

$$\{[1]\} \cup \{[1,3,\dots,i]|3 \le i \le r\} \cup \{[i,\dots,j]|2 \le i \le j \le r\} \cup \{[j,\dots,2,1,3,\dots,k]|2 \le j < k \le r\}.$$

1.2. **Root Vectors.** Now, calculate the root vectors:

Proposition 1.2.1. *In type A_r,*

$$b^*[i,\dots,j] = [i,\dots,j].$$

Proposition 1.2.2. *In type B_r:*

$$b^*[i,\dots,j] = [i,\dots,j]$$
$$b^*[j,\dots,1,1,\dots,k] = (2)_0[j,\dots,1,1,\dots,k].$$

Proposition 1.2.3. *In type C_r:*

$$b^*[i,\dots,j] = [i,\dots,j]$$
$$b^*[j,\dots,2,1,2,\dots,k] = [j,\dots,2,1,2,\dots,k]$$
$$b^*[1,\dots,j,2,\dots,j] = [1]([2,\dots,j] * [2,\dots,j]).$$

Proposition 1.2.4. *In type D_r:*

$$\begin{aligned} b^*[1] &= [1] \\ b^*[1,3,\dots,i] &= [1,3,\dots,i] \\ b^*[i,\dots,j] &= [i,\dots,j] \\ b^*[2,1,3,\dots,k] &= [1,2,3,\dots,k] + [2,1,3,\dots,k] \\ b^*[j,\dots,2,1,3,\dots,k] &= [j,\dots,3,2,1,3,\dots,k] + [j,\dots,3,1,2,3,\dots,k]. \end{aligned}$$

Department of Mathematics, University of California, Berkeley, Berkeley, CA 94720-3840
E-mail address: dhill1@math.berkeley.edu