Math 1a – Quiz 8

SOLUTIONS

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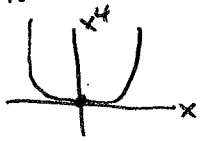
1. (1 point) True or false: If $f(x)$ is a twice differentiable function and $f''(c) = 0$, then f has an inflection point at $x = c$.

False. An inflection point is where f changes concavity. If f is twice differentiable, then $f''=0$ at an inflection pt. However, $f(x) = x^4$ has $f''(0) = 0$, but 0 is not an inflection pt.



2. (2 points) Evaluate the following limits.

(a) $\lim_{t\to 1} \frac{\ln t}{(t-1)^3}$ type $\frac{0}{0}$

L'Hôpital: $\lim_{t\to 1} \frac{1/t}{3(t-1)^2} = \lim_{t\to 1} \frac{1}{3t(t-1)^2} = \frac{1}{0} \longrightarrow \pm\infty$, or DNE.

Denominator is always positive near -1, so limit is $\boxed{\infty}$

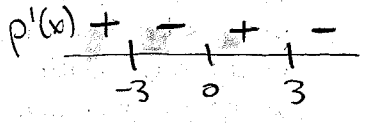
(b) $\lim_{\theta\to\pi} \frac{\sin(\theta/2)}{\theta}$

Don't need l'Hôpital!! $\lim_{\theta\to\pi} \frac{\sin\theta/2}{\theta} = \frac{\sin \pi/2}{\pi} = \boxed{\frac{1}{\pi}}$

3. (4 points) Let $p(x) = -x^4 + 18x^2 - 12$.

(a) For what values of x is $p(x)$ increasing?

$p'(x) = -4x^3 + 36x = -4x(x^2-9)$



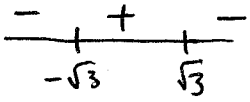
p is increasing where $p' > 0$:

$\boxed{(-\infty, -3) \text{ and } (0, 3)}$

(b) For what values of x is $p(x)$ concave down?

$p(x)$ is concave down where $p''(x) < 0$

$p''(x) = -12x^2 + 36 = -12(x^2-3)$



So p is concave down on

$\boxed{(-\infty, -\sqrt{3}) \text{ and } (\sqrt{3}, \infty)}$

4. (5 points) Consider the function $f(x) = 3^x + x^3$.

(a) Show that $f(x)$ has *at most* one zero.

If f had two zeroes, then by Rolle's theorem $f'(x)$ would have a zero. But $f'(x) = 3^x \ln 3 + 3x^2$.

$3^x \ln 3 > 0$ and $3x^2 \geq 0$ for all x, so $f'(x) > 0$ for all x.

Thus f has at most one zero.

(b) Use the Intermediate Value Theorem to show that $f(x)$ has *at least* one zero.

$f(0) = 3^0 + 0^3 = 1$

$f(-1) = 3^{-1} + (-1)^3 = -\frac{2}{3}$

} f is continuous, so by IVT f has a zero between -1 and 0.

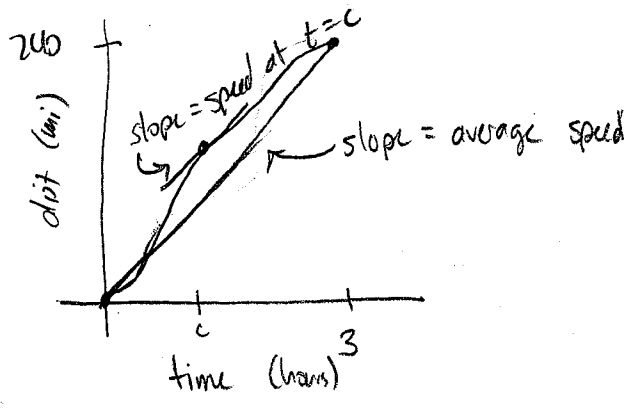
(c) Show that $f(x)$ has *at most* two zeroes.

We showed in (a) that f has at most one zero (i.e. ≤ 1), so it has to have at most 2 (≤ 2)!

(d) How many zeroes does $f(x)$ have?

At most one + at least one = exactly one zero.

5. (3 points) You're cruising down I-5 between San Francisco and Los Angeles, where the speed limit is 70 miles per hour. At 3 pm, a police officer notes your position and license plate number and enters it into a database. Three hours later, a different police officer spots you 240 miles farther south, looks up your information, and pulls you over. Use the Mean Value Theorem to explain why you are getting a speeding ticket even though neither officer caught you going over the speed limit. (Hint: a graph of your position versus time might be helpful. Don't assume you traveled at a constant speed!)



Let $f(t)$ be the distance traveled at time t. Then $f(0) = 0$, $f(3) = 240$, and f is continuous and differentiable. The Mean Value Theorem says that there is a c between 0 and 3 such that

$$f'(c) = \frac{f(3) - f(0)}{3 - 0} = 80 \text{ miles/hour}$$

Thus at time c, your instantaneous speed was 80 mph, above the speed limit.