Math 1a – Quiz 3

SOLUTIONS

GSI: David Freeman

September 15, 2005

1. (2 points each). Compute the following six limits.

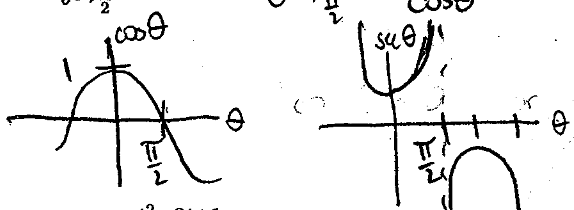
(a) $\lim_{h\to 0} \frac{(2+h)^3-8}{h} = \lim_{h\to 0} \frac{(8+12h+6h^2+h^3)-8}{h}$

$= \lim_{h\to 0} \frac{12h+6h^2+h^3}{h} = \lim_{h\to 0} 12+6h+h^2 = \boxed{12}$

(b) $\lim_{y\to 7} \frac{\sqrt{y+2}-3}{y-7} = \lim_{y\to 7} \left(\frac{\sqrt{y+2}-3}{y-7}\right)\left(\frac{\sqrt{y+2}+3}{\sqrt{y+2}+3}\right)$

$= \lim_{y\to 7} \frac{y+2-9}{(y-7)(\sqrt{y+2}+3)} = \lim_{y\to 7} \frac{1}{\sqrt{y+2}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$

(c) $\lim_{\theta\to\frac{\pi}{2}^+} \sec\theta = \lim_{\theta\to\frac{\pi}{2}^+} \frac{1}{\cos\theta}$

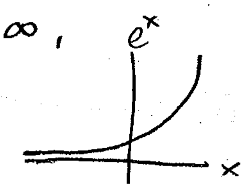


$\cos\theta < 0$ as θ approaches $\frac{\pi}{2}$ from the right, so the limit is $\boxed{-\infty}$

(d) $\lim_{t\to\infty} \frac{e^{t^2-3t+1}}{e^{t^3-t+2}}$

$= \lim_{t\to\infty} e^{-t^3+t^2-2t-1}$ (exponent laws)

$-t^3+t^2-2t-1 \to -\infty$ as $t\to\infty$, so the limit is $\boxed{0}$



(e) $\lim_{x\to 2^+} \left(\frac{3}{|x-2|} + \frac{3}{x-2}\right)$ for $x-2>0$, $|x-2| = x-2$, so

the limit is $\lim_{x\to 2^+} \left(\frac{3}{x-2} + \frac{3}{x-2}\right) = \lim_{x\to 2^+} \frac{6}{x-2} = \boxed{\infty}$

(f) $\lim_{x \to 2^-} \left(\frac{3}{|x-2|} + \frac{3}{x-2} \right)$ $|x-2| = -(x-2)$ when $x - 2 < 0$, so

the limit is $\lim_{x \to 2^-} \left(\frac{3}{-(x-2)} + \frac{3}{x-2} \right) = \lim_{x \to 2^-} 0 = \boxed{0.}$

2. (3 points). Sketch the graph of an example of a function f that satisfies all of the given conditions.

(a) $\lim_{x \to 0^-} f(x) = 1$

(b) $\lim_{x \to 0^+} f(x) = \infty$

(c) $\lim_{x \to -\infty} f(x) = -1$

(d) $\lim_{x \to \infty} f(x) = -\infty$

(e) $\lim_{x \to -1} f(x) = 0$

(f) f is defined at -1, and $f(-1) \neq 0$.

One solution: