Math 1a – Quiz 12

SOLUTIONS

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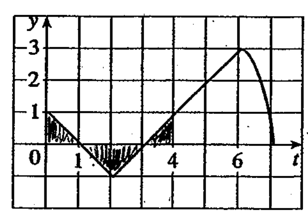
1. (1 point) If $f(x)$ is the slope of a trail at a distance of x miles from the start of the trail, what does $\int_3^5 f(x)\,dx$ represent?

Slope = rate of change of the trail's height.

$\int_3^5 f(x)\,dx$ = Net change in trail's height between 3 & 5 miles from the start.
("height" could also be "altitude" or "vertical displacement")

2. (5 points) Let f be the function graphed below. Let $g(x) = \int_0^x f(t)\,dt$.

Fundamental thm: $g'(x) = f(x)$.



(a) What is $g(4)$?

$g(4)$ = (area between 0 & 1) − (area between 1 & 3) + (area between 3 & 4) $= 0$.

(b) Where does g have a maximum value?

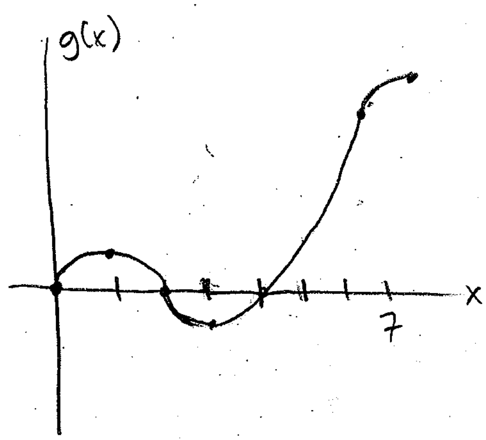
$g'(x) = f(x) = 0$ when $x = 1, 3, 7$; also check $x = 0$.
at $\boxed{x=7}$ the area under the graph is the greatest

(c) Where does g have a minimum value?

$g'(x)$ goes from neg. to pos. at $x = 3$, so $x = 3$ is local min.
$g(3) < 0$, $g(0) > 0$, so $\boxed{x=3}$ is the minimum.

(d) Sketch a rough graph of g.

Use curve sketching techniques:
- $g(0) = g(2) = g(4) = 0$
- local max at $x=1$, $x=7$
- local min at $x=0$, $x=3$
- inflection pts at $x=2$, $x=6$ since g'' goes from positive to negative



3. (2 points) Which of the following is equal to $\int 2\tan x \sec^2 x \, dx$?

(a) $\tan^2 x + C$

(b) $\sec^2 x + C$

(c) Both (a) and (b).

(d) None of the above.

take derivatives:

$\frac{d}{dx}(\tan^2 x + C) = 2\tan x \cdot \sec^2 x$

$\frac{d}{dx}(\sec^2 x + c) = 2\sec x \cdot \sec x \tan x$

(this works because $\sec^2 x = \tan^2 x + 1$ and 2 functions that differ by a constant have the same derivative.)

4. (4 points) Let $f(x) = \int_1^{\sqrt{x}} t^4 \, dt$.

(a) Compute $f(4)$.

$$f(4) = \int_1^2 t^4 dt = \frac{t^5}{5}\Big|_{t=1}^{2} = \frac{2^5}{5} - \frac{1^5}{5} = \boxed{\frac{31}{5}}$$

(b) Compute $f'(4)$.

$$f(x) = \int_1^{\sqrt{x}} t^4 dt = \frac{t^5}{5}\Big|_1^{\sqrt{x}} = \frac{x^{5/2}}{5} - \frac{1}{5}$$

so $$f'(x) = \frac{5}{2} \cdot \frac{x^{3/2}}{5} = \frac{x^{3/2}}{2}$$

$$f'(4) = \frac{4^{3/2}}{2} = \frac{2^3}{2} = \boxed{4}$$

5. (3 points) Compute $\int_{-2}^{2} |x-1| \, dx$.

Algebraically: $|x-1| = \begin{cases} x-1 & x \geq 1 \\ -(x-1) & x \leq 1 \end{cases}$

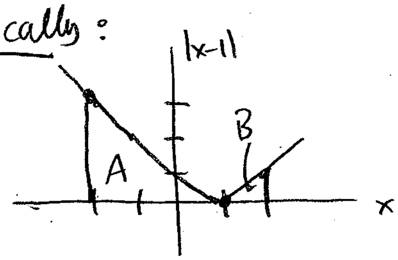
so $$\int_{-2}^{2} |x-1| dx = \int_{-2}^{1} (-x+1) dx + \int_1^2 (x-1) dx$$

$$= \left(-\frac{x^2}{2} + x\right)\Big|_{-2}^{1} + \left(\frac{x^2}{2} - x\right)\Big|_1^2$$

$$= \left(-\frac{1}{2}+1\right) - \left(-\frac{4}{2} - 2\right) + \left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right)$$

$$= \frac{1}{2} + 4 + 0 + \frac{1}{2} = \boxed{5}$$

Graphically:



area $A = \frac{1}{2}(3)(3) = \frac{9}{2}$

area $B = \frac{1}{2}(1)(1) = \frac{1}{2}$

total area $= \boxed{5}$