

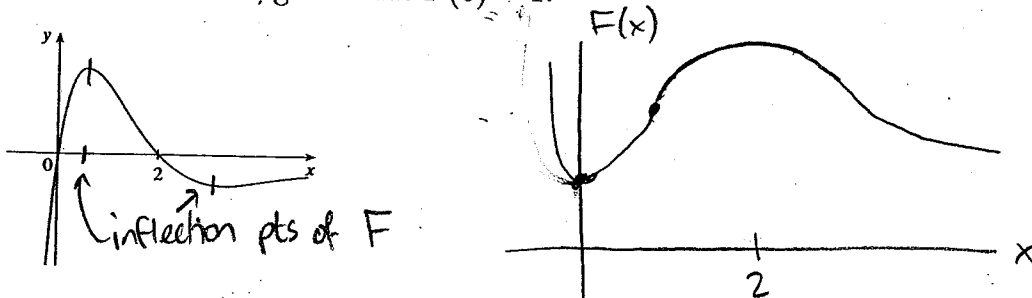
Math 1a - Quiz 10

SOLUTIONS

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1. (3 points) The graph of a function is shown in the figure. Make a rough sketch of an antiderivative F , given that $F(0) = 1$.



$F' = 0$ at $x=0, 2$

local min at $x=0$, local max at $x=2$

2. (4 points) If $f'(x) = \sin x + \frac{3}{\sqrt{x}} - 2$, and $f(0) = 3$, find a formula for $f(x)$.

$$f'(x) = \sin x + 3x^{-1/2} - 2$$

$$f(x) = -\cos x + \frac{3x^{1/2}}{1/2} - 2x + C$$

$$= -\cos x + 6x^{1/2} - 2x + C$$

$$f(0) = -1 + 0 - 0 + C = 3, \text{ so } C = 4$$

$$f(x) = -\cos x + 6x^{1/2} - 2x + 4$$

3. (4 points) Let $\alpha = \ln 10$. Suppose we want to use Newton's Method to compute a decimal approximation to α .

α is a number, not a variable.

- (a) Write down an equation $f(x)$ that has α as a zero.

The question is vague, but I was looking for

$$f(x) = e^x - 10$$

We want to use Newton's Method to compute

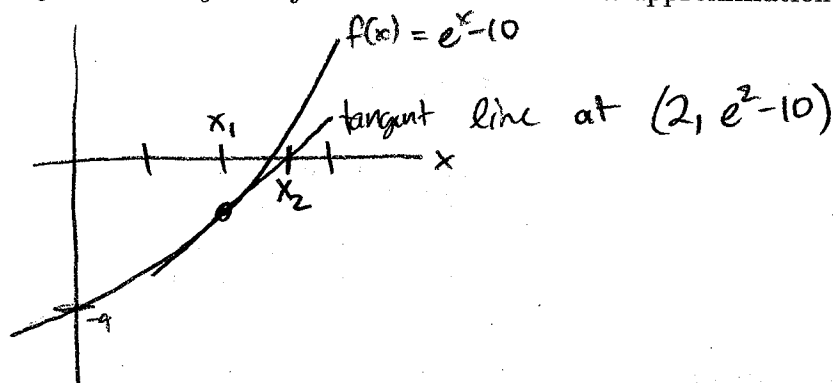
If you write an equation $\ln 10$, so an equation with $\ln 10$ in it won't help much.

- (b) Which of the following would be the best first guess for finding a zero of $f(x)$ using Newton's Method: 0, 1, 2, or 10?

$\ln 10$ is between 2 and 3, so

the best guess is $\boxed{2}$

- (c) If x_1 is your first guess from part (b), draw a picture using the graph of $f(x)$ that illustrates *geometrically* how you would find the next approximation x_2 .

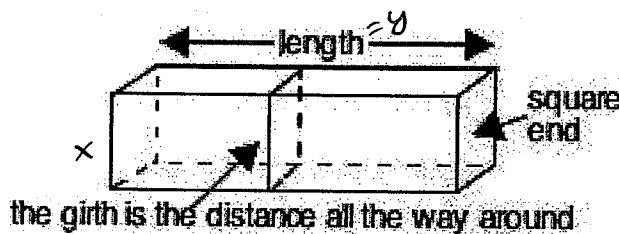


- (d) If x_1 is your first guess from part (b), compute x_2 *algebraically*. You don't need to simplify your answer.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \left| \quad \begin{array}{l} f(x) = e^x - 10 \\ f'(x) = e^x \end{array} \right.$$

$$x_2 = 2 - \frac{e^2 - 10}{e^2}$$

4. (4 points) The U.S. Postal Service will accept a box for domestic shipment only if the sum of the length and girth (distance around) does not exceed 72 inches. Find the dimensions of the largest acceptable box with a square end.



label length = y , side of square end = x

$$4x + y = 72$$

$$V = x^2 y \quad \leftarrow \text{maximize volume}$$

$$\text{Substitute: } y = 72 - 4x$$

$$V = x^2(72 - 4x) = 72x^2 - 4x^3$$

$$\frac{dV}{dx} = 144x - 12x^2 = 12x(12 - x)$$

$$x = 0, \text{ or } 12.$$

$$V = 0 \text{ when } x = 0, \quad 2$$

so $x = 12$ is maximum.

$$\text{dim length } y = 72 - 4 \cdot 12 = 24$$

optimal dimensions are

$$\boxed{12 \times 12 \times 24 \text{ inches}}$$