Math 1a – Practice Midterm 2

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Part I

Each question is worth 2 points.

1. Suppose f and g are increasing functions on an interval I. Which one of the following must be true?

   (a) $f(x) > 0$ for all x in I.

   (b) $g''(x) > 0$ for all x in I.

   (c) fg is increasing on I.

   (d) $f - g$ is decreasing on I.

   (e) $1/f$ is increasing on I.

   (f) g^3 is not decreasing on I.

2. Suppose $f(x)$ is defined and twice differentiable for all real x. Which one of the following could be a possible description of f and its first and second derivatives?

   (a) $f(x) < 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x.

   (b) $f(x)$ has two local minima and $f''(x)$ has three zeroes.

   (c) $f(1) = -2$, $f(3) = 0$, and $f'(x) > 0$ for all x.

   (d) $f(x)$ has two local maxima and $f''(x)$ has one zero.

   (e) $f(-1) = 0$, $f(0) = -1$, $f''(x) < 0$, and $f'(x) \leq -1$ for all x.

   (f) $f(x)$ has three zeroes and no local maxima.

Part II

Each question is worth 3 points.

1. Answer all of the following.

   (a) Find a function $f(x)$ that is defined for all $x > 0$, has $f(1) = 1$, and has $f'(x) = 1/x$.

   (b) Differentiate the following with respect to x:

   $$\frac{dy}{dx} \cos y.$$

   (c) Compute the derivative of $f(x) = x^{e^x}$.

2. Compute the following limits.

   (a) $\lim_{x \to 0} \dfrac{\cosh x - 1}{x^2}$

   (b) $\lim_{x \to \infty} \left(1 + \dfrac{3}{x}\right)^x$

   (c) $\lim_{x \to 0^+} \dfrac{1}{x} + \ln x$

3. Let $f(x)$ be the following function, defined for all positive real numbers:

   $$f(x) = \begin{cases} 3 + \ln x & 0 < x < 1 \\ 3\cos(\pi x/2) & 1 \leq x \leq 5 \\ \tan^{-1} x & x > 5. \end{cases}.$$

   If x is your answer to this question, then your score is $f(x)$, rounded down to the nearest integer if necessary. If $f(x)$ is negative you will get a score of zero.

4. Sand falls from a conveyor belt at a rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always equal to the diameter of the base. How fast is the height changing when the pile is 4 m high?

5. Sketch a graph of the function $y = e^{-x^2}$.