I. Exam A: (b), Exam B: (c)
The function is increasing and concave down at $x = a$, so $f'(a) > 0$ and $f''(a) < 0$.

II. Exam A: (d), Exam B: (b)
The product rule gives $f'(x) = x \cdot \frac{d}{dx}\sqrt{x^3+7} + \frac{d}{dx}x \cdot \sqrt{x^3+7}$. Applying the chain rule (a.k.a. the general power rule) to the first term gives $f'(x) = \frac{x \cdot 3x^2}{\sqrt{x^3+7}} + \sqrt{x^3+7}$. Getting a common denominator gives

$$f'(x) = \frac{3x^3 + 2\sqrt{x^3+7}\sqrt{x^3+7}}{2\sqrt{x^3+7}} = \frac{5x^3+14}{2\sqrt{x^3+7}}$$

.

III. Exam A: (a), Exam B: (d)
The quotient rule gives

$$f'(x) = \frac{(x^2-1)(2x) - (2x)(x^2+5)}{(x^2-1)^2} = \frac{-12x}{(x^2-1)^2}$$

IV. Exam A: (d), Exam B: (a)
The quotient rule gives

$$f'(x) = \frac{e^t \cdot 1 - (t+1)e^t}{(e^t)^2} = -\frac{te^t}{(e^t)^2} = -\frac{t}{e^t}$$

V. This is the only question where the problem itself was different on the two exams.

Exam A: (b) (c) (b) (b) (a)

(1) We have $f'(x) = x^3 - x^2 - 2x$, which factors as $x(x-2)(x+1)$, so there are critical points at $x = -1$, 0, and 2. We also have $f''(x) = 3x^2 - 2x - 2$. Plugging in the values of the critical points, we see $f''(-1) > 0$, $f''(0) < 0$, and $f''(2) > 0$. Thus there are local minima at $x = -1$ and $x = 2$.

(2) To see which local minimum is a global minimum we must check the value of the function: $f(-1) = -5/12$, and $f(2) = -8/3$, so $x = 2$ is the global minimum.

(3) We have $f'(1) = -2$, so the function is decreasing at $x = 1$.

(4) We have $f''(1) = -1$, so there is no inflection point at $x = 1$. (An inflection point must have $f''(x) = 0$.)

(5) Graph (a) is has local minima at $x = -1$ and $x = 2$, and a global minimum at $x = 2$.

Exam B: (c) (d) (b) (b) (b)

(1) We have $f'(x) = -x^4 - x^3 + 2x$, which factors as $-x(x+2)(x-1)$, so there are critical points at $x = 1$, 0, and -2. We also have $f''(x) = -3x^2 - 2x + 2$. Plugging in the values of the critical points, we see $f''(1) < 0$, $f''(0) > 0$, and $f''(-2) < 0$. Thus there are local maxima at $x = 1$ and $x = -2$.

(2) To see which local maximum is a global maximum we must check the value of the function: $f(1) = 5/12$, and $f(-2) = 8/3$, so $x = -2$ is the global minimum.

(3) We have $f''(1) = -3$, so there is no inflection point at $x = 1$. (An inflection point must have $f''(x) = 0$.)

(4) We have $f'(1) = 0$, so the function is neither increasing nor decreasing at $x = 1$.

(5) Graph (b) is has local maxima at $x = 1$ and $x = -2$, and a global maximum at $x = -2$.

VI. Exam A: (d) (c), Exam B: (a) (d)

(1) $f(g(x)) = (x^2+1) + (x^2+1)^2 = x^4 + 3x^2 + 2$.

(2) By the chain rule, $\frac{du}{dx} = \frac{dz}{dx} + 2z\frac{dz}{dx}$. Substituting $\frac{dz}{dx} = 2x$ gives

$$\frac{du}{dx} = 2x + 2(x^2+1)(2x) = 2x(1 + 2(x^2+1)).$$

You could also differentiate the answer to part 1 to get $\frac{du}{dx} = 4x^3 + 6x$, and multiply out the three choices to see which matches.

VII. Exam A: (d) (b) (b), Exam B: (c) (a) (d)

(1) We have the point $x = 1000$, $p = 50$. If the fare increases to \$60 then the number of passengers decreases to 950, so this gives the point $x = 950$, $p = 60$. The slope for the line connecting these two points is $\frac{60-50}{950-1000} = -\frac{1}{5}$. Plugging the first point into point-slope form gives the equation $p - 50 = -\frac{1}{5}(x - 1000)$. Multiplying both sides by 5 and moving things around gives $x + 5p = 1250$.

(2) Revenue is (cost per passenger) $\times$ (number of passengers) $= px = p(1250 - 5p) = 1250p - 5p^2$.

(3) The derivative of revenue with respect to p is $1250 - 10p$; this is zero when $p = 125$.

VIII. Exam A: (c) (a), Exam B: (b) (b)

(1) Differentiating the equation implicitly (using the product rule) gives

$$(x \cdot 5y^4 \frac{dy}{dx} + 1 \cdot y^5) + (x^3 \cdot \frac{dy}{dx} + 3x^2 \cdot y) = 0.$$

Solving for $\frac{dy}{dx}$ gives

$$\frac{dy}{dx} = -\frac{3x^2y + y^5}{5xy^4 + x^3}.$$

(2) At $x = 1$, the equation from part 1 gives $\frac{dy}{dx} = -\frac{3+1}{5+1} = -\frac{2}{3}$.

IX. Exam A: (b) (c) (a), Exam B: (a) (b) (c)

(1) Since car B is moving towards P, the value of y is decreasing, so the derivative $y'(t)$ is negative.

(2) The Pythagorean Theorem gives $z(t)^2 = x(t)^2 + y(t)^2$. Differentiating with respect to t using the chain rule gives $2z(t)z'(t) = 2x(t)x'(t) + 2y(t)y'(t)$. Solving for $z'(t)$ gives

$$z'(t) = \frac{1}{z(t)}(x(t)x'(t) + y(t)y'(t)).$$

(3) If $x = 4$ and $y = 3$, then $z = 5$. We also have $x'(t) = -10$ and $y'(t) = -15$ (the negative signs come from the fact that both quantities are decreasing). The formula from part 2 now gives $z'(t) = \frac{1}{5}(4(-10) + 3(-15)) = -17$.