

Disclaimer!

This review sheet contains more questions than you could reasonably be expected to solve in 3 hours.

The questions are for your own practice and are intended to indicate the expected scope and difficulty of the final exam.

The questions and topics on the actual final exam may differ significantly from the questions below.

1. Which of the following integrals is approximated by the Riemann sum

$$\frac{1}{2}e + \frac{1}{2}e^{1.5} + \frac{1}{2}e^2 + \frac{1}{2}e^{2.5} ?$$

- (a) $\int_1^{2.5} \frac{1}{2}e^x dx$
(b) $\int_1^3 \frac{1}{2}e^x dx$
(c) $\int_1^{2.5} e^x dx$
(d) $\int_1^3 e^x dx$
2. What is $\int xe^{x^2} dx$?
- (a) $\frac{1}{3}e^{x^3} + C$
(b) $2x^2e^{x^2} + e^{x^2} + C$
(c) $\frac{1}{2}e^{x^2} + C$
(d) $\frac{1}{2}x^2 + e^{x^2} + C$
3. What is $\lim_{h \rightarrow 0} \frac{e^{-2h} - 1}{h}$?
- (a) 0
(b) 1
(c) 2
(d) -2
(e) Does not exist.

4. An 8 gram sample of a radioactive material was put into storage in 1980. When the sample was examined in 2000, 2 grams of the material remained. How much material was there in 1990?
- (a) 3 grams
 - (b) 4 grams
 - (c) 5 grams
 - (d) 6 grams

Questions 5–7 relate to the following situation: You find yourself at point A on a 40-meter high catwalk, in possession of a 16-ton weight, with your evil nemesis directly below you. You decide to seize the opportunity and drop the weight. You don't want to get caught, so immediately after dropping the weight you start to sprint along the catwalk at a rate of 3 meters per second. Suppose the velocity of the weight after t seconds is given by $y'(t) = -10t$ meters per second.

5. What is the height $y(t)$ of the 16-ton weight (in meters) after t seconds?
- (a) $y(t) = -5t^2 + 40$
 - (b) $y(t) = 5t^2 - 40$
 - (c) $y(t) = 5t^2 + 40$
 - (d) $y(t) = 10t - 40$
6. If $x(t)$ is your distance from point A and $z(t)$ is the distance between you and the weight, which of the following is true?
- (a) $z(t) = x(t) + y(t) - 40$
 - (b) $z(t)^2 = x(t)^2 + y(t)^2$
 - (c) $z(t)^2 = x(t)^2 + (40 - y(t))^2$
 - (d) $z(t)^2 = x(t)^2 - (40 - y(t))^2$
7. How fast is the distance between you and the weight increasing two seconds after you drop it?
- (a) $403/\sqrt{436}$ m/s
 - (b) $406/\sqrt{436}$ m/s
 - (c) $418/\sqrt{436}$ m/s
 - (d) $406/\sqrt{409}$ m/s

8. What is $\lim_{x \rightarrow \infty} \frac{3x^2 + 2e^x}{2x^3 + e^x}$
- (a) 0
 - (b) $\frac{3}{2}$
 - (c) 2
 - (d) Does not exist
9. What is the derivative of $(\ln x)^3$?
- (a) $3(\ln x)^2$
 - (b) $\frac{1}{(\ln x)^3}$
 - (c) $\frac{1}{x(\ln x)^3}$
 - (d) $\frac{3(\ln x)^2}{x}$
 - (e) $\frac{3}{x}$
10. Which of the following is equal to $3 \ln 5$?
- (a) $\frac{3}{2} \ln 25$
 - (b) $(\ln 5)^3$
 - (c) $\ln \frac{1}{125}$
 - (d) $\ln 15$
11. What is the area of the region between $f(x) = x^2 + 2x + 18$ and $g(x) = -x^2 + 14x + 8$?
- (a) $\frac{-64}{3}$
 - (b) -12
 - (c) $\frac{64}{3}$
 - (d) 12
12. The derivative of $x^{2/3}y^{3/2} = 1$ at the point $(2^9, 1/2^4)$ is
- (a) -2^{13}
 - (b) $-2^{15}/9$
 - (c) $-\frac{1}{3^2 2^{13}}$
 - (d) None of the above.

13. The tangent line to xe^x at $(2, 2e^2)$ has equation
- (a) $y - 2e^2 = 2e^2(x - 2)$
 - (b) $y = 3e^2x - 4e^2$
 - (c) $y = 2e^2x - 4e^2$
 - (d) $y = 3e^2x - 8e^2$
14. T or F: if $2 \leq f(x) \leq 3$ whenever $1 \leq x \leq 7$ then $12 \leq \int_1^7 f(x)dx \leq 18$.
15. Let $F(x)$ be a function whose derivative is $4/x$. Suppose $F(x)$ goes through the point $(1, \ln 3)$. What is $F(2)$?
- (a) $\ln 11$
 - (b) $\ln 19$
 - (c) $\ln 24$
 - (d) $\ln 48$

From chemistry, we know that pressure P , volume V and temperature T of a quantity of gas are related by the equation $PV = kT$, where k is a constant. Use this equation to answer questions 16 and 17.

16. For a gas in a box, volume is a constant and pressure and temperature can change. What is the equation of the rate of change of pressure as temperature changes?
- (a) $dP/dT = k/V$
 - (b) $dT/dP = (V + P)/k$
 - (c) $dP/dT = kV$
 - (d) $dP/dx = kT/V$
17. For a gas in a balloon, suppose temperature stays constant and pressure and volume can change. What is dP/dV ?
- (a) $(kT - P)/V$
 - (b) $-P/V$
 - (c) $-P/V^2$
 - (d) $(kT - P)/V^2$

18. Which of the following is the Riemann sum approximating the area under $f(x) = xe^x$ for $2 \leq x \leq 5$, by dividing it into 6 intervals and evaluating at right endpoints?
- (a) $(5/2e^{5/2} + 3e^3 + 7/2e^{7/2} + 4e^4 + 9/2e^{9/2} + 5e^5)$
 - (b) $(2e^2 + 5/2e^{5/2} + 3e^3 + 7/2e^{7/2} + 4e^4 + 9/2e^{9/2})$
 - (c) $1/2(5/2e^{5/2} + 3e^3 + 7/2e^{7/2} + 4e^4 + 9/2e^{9/2} + 5e^5)$
 - (d) $1/2(2e^2 + 5/2e^{5/2} + 3e^3 + 7/2e^{7/2} + 4e^4 + 9/2e^{9/2})$
19. If an object's acceleration is given by $a(t) = 4 \text{ m/s}^2$ and its initial velocity is $v(0) = -5 \text{ m/s}$, what is the object's average velocity between time $t = 0$ and $t = 10$?
- (a) 15 m/s
 - (b) 20 m/s
 - (c) 25 m/s
 - (d) 35 m/s

20. Find

$$\frac{d}{dx} \left(\frac{(x-2)^2}{(x-3)^2} \right)$$

- (a) $\frac{-10(x-2)}{(x-3)^3}$
 - (b) $\frac{-2(x-2)}{(x-3)^3}$
 - (c) $\frac{x-2}{x-3}$
 - (d) $2\frac{x-2}{x-3}$
21. Consider the graph of $y = e^x$. Which of the following expressions gives the volume of the solid obtained by revolving the portion of the graph from $x = 0$ to $x = 2$ around the x -axis?
- (a) $\int_0^2 \pi e^x dx$
 - (b) $\int_0^2 \pi e^{2x} dx$
 - (c) $\int_0^2 \pi e^{x^2} dx$
 - (d) $\int_0^2 \pi e^{2x^2} dx$

22. What is the volume of the solid described in problem 21?

- (a) $\frac{\pi}{2}(e^2 - 1)$
- (b) $\frac{\pi}{2}e^4$
- (c) $\frac{\pi}{2}(e^4 - 1)$
- (d) $\frac{\pi}{4}(e^2 - 1)$

23. What is the sum of the x -values at which $f(x) = x^2e^x$ has an inflection point?

- (a) -4
- (b) $-2\sqrt{2}$
- (c) -2
- (d) 0

24. For $f(x) = \ln[(x^2 + 1)^2]$ what is $f'(1)$?

- (a) $\frac{1}{2}$
- (b) 1
- (c) 2
- (d) 4

25. For

$$f(x) = \begin{cases} x^2 - 1, & \text{if } x < 1 \\ 2x - 2, & \text{if } 1 \leq x \leq 2 \\ x^3 - 10x, & \text{if } x > 2 \end{cases}$$

where does $f'(x)$ not exist?

- (a) Exists Everywhere
- (b) $x = 1$
- (c) $x = 2$
- (d) $x = 1$ and $x = 2$

26. What is the area between the two curves $y = x^2$ and $y = x^3$ between $x = 0$ and $x = 2$?

- (a) $-\frac{4}{3}$
- (b) $\frac{4}{3}$
- (c) $\frac{3}{2}$
- (d) $\frac{11}{6}$

27. What is the value of $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{e^x - 1}$

- (a) $e - 1$
- (b) $e + 1$
- (c) 2
- (d) Does not exist

28. How many points of relative extremum does $f(x) = e^x - \ln|x|$, $x \neq 0$ have?

- (a) 0
- (b) 1
- (c) 2
- (d) Infinitely Many

29. What does $\frac{8^{3/2} \cdot 32^{1/3}}{4^{4/3}}$ simplify to?

- (a) $4\sqrt{2}$
- (b) 8
- (c) $8\sqrt{2}$
- (d) 16

30. If you are investing money in a trust which pays five percent interest compounded continuously, which of the following investments will give you the largest amount at its completion? (Hint: you may want to use the fact that $e^{0.5} \approx 1.6$.)

- (a) \$10,000 invested for 50 years
- (b) \$20,000 invested for 40 years
- (c) \$30,000 invested for 30 years
- (d) \$40,000 invested for 20 years

31. What is the value of $\int_2^4 \frac{t+t^{-2}}{2} dt$

- (a) $\frac{25}{8}$
- (b) $\frac{13}{4}$
- (c) 4
- (d) $\frac{49}{8}$

32. Suppose there is an open rectangular box (no top) with square base that has volume 500 m^3 . If we choose the box of this type which has minimum surface area, what is the sum of the three dimensions of the box?

- (a) 15 m
- (b) 25 m
- (c) 35 m
- (d) 45 m

33. What is the largest value of x satisfying the equation $\ln(x^2 + \frac{2e^4}{9}) = \ln x + 2$

- (a) $\frac{e}{3}$
- (b) $\frac{2e}{3}$
- (c) $\frac{e^2}{3}$
- (d) $\frac{2e^2}{3}$

34. Which of the following functions' derivative exists at $x = 0$?

- (a) $f(x) = |x|$
- (b) $f(x) = |x^2 - x|$
- (c) $f(x) = |x^2 + x|$
- (d) $f(x) = |x^3 + x^2|$

35. What is the value of $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{8x^3 + 1}}{x + 4}$

- (a) 1
- (b) 2
- (c) 8
- (d) Does not exist

36. If we have 80 g of a substance with a half-life of 10 years, after how many years will only 10 g of the substance remain?

- (a) 20
- (b) 30
- (c) 40
- (d) 50

37. For the curve defined by $xy^2 + x^2y^4 = 2x^3$, what is the value of $\frac{dy}{dx}(1, 1)$?
- (a) $\frac{1}{2}$
 - (b) 1
 - (c) 2
 - (d) 4
38. How many horizontal asymptotes does the function $f(x) = \frac{1}{1+e^x}$ have?
- (a) 0
 - (b) 1
 - (c) 2
 - (d) 3
39. What is larger: the area under $y = \frac{1}{x}$ between $x = 2$ and $x = 6$ or the Riemann sum approximation for that area which uses the right endpoints and $n = 4$?
- (a) The actual area
 - (b) The Riemann Sum
 - (c) They are the same
40. For $f(x) = (x^2 + 3x + 1)^5$, $f'(0)$ is
- (a) 3
 - (b) 5
 - (c) 15
 - (d) 45
41. The function $f(x) = \frac{x^2}{2}\ln x - 2x\ln x - \frac{x^2}{4} + 2x$ (defined for $x > 0$) has which of the following?
- (a) One Local Maximum
 - (b) One Local Maximum and One Local Minimum
 - (c) Two Local Maximums
 - (d) Two Local Minimums
42. Let $f(x)$ be the function $\frac{x^4}{4} - \frac{x^2}{2}$ defined on the interval $-1 \leq x \leq 2$. At what value of x does the absolute maximum of $f(x)$ occur?
- (a) -1
 - (b) 0
 - (c) 1
 - (d) 2

43. What does $e^{2\ln 3 - 3\ln 2}$ simplify to?
- (a) $\frac{3}{4}$
 - (b) 1
 - (c) $\frac{9}{8}$
 - (d) 2
44. How many times do the curves $y = x^3 - x^2 + 5x + 4$ and $y = 3x^3 + 3x^2 + 7x + 4$ intersect?
- (a) 0
 - (b) 1
 - (c) 2
 - (d) 3
45. What is the equation of the tangent line to $f(x) = \frac{x^2+1}{x-1}$ at $x = 2$?
- (a) $y = -x + 7$
 - (b) $y = x + 3$
 - (c) $y = -x + 1$
 - (d) $y = x - 3$
46. Which of the following functions is concave up everywhere?
- (a) $f(x) = x$
 - (b) $f(x) = x^3$
 - (c) $f(x) = \ln x, x > 0$
 - (d) $f(x) = e^{x^2}$
47. If the length of a rectangle is increasing at a rate of 2 ft/s, at what rate should the width be changing so that the area is not changing when the rectangle has length 5 ft and width 10 ft?
- (a) -4 ft/s
 - (b) -2 ft/s
 - (c) -1 ft/s
 - (d) $\frac{1}{2}$ ft/s
48. What is the volume of the solid created by rotating the portion of the curve $y = x - x^2$ in the first quadrant about the x -axis?
- (a) $\frac{\pi}{30}$
 - (b) $\frac{\pi}{15}$
 - (c) $\frac{2\pi}{15}$
 - (d) $\frac{\pi}{6}$

49. For $f(x) = (x^2 + 1)^x$, $f'(0)$ is equivalent to which of the following

- (a) $\lim_{h \rightarrow 0} \frac{(h^2 + 1)^h}{h}$
- (b) $\lim_{h \rightarrow 0} \frac{(h^2 + 1)^h - 1}{h}$
- (c) $\lim_{h \rightarrow 0} \frac{\ln[(h^2 + 1)^h]}{h}$
- (d) $\lim_{h \rightarrow 0} \frac{[(h + 1)^2 + 1]^{h+1} - 1}{h}$

50. What is the area under the curve $y = \sqrt{x}$ from $x = 4$ to $x = 9$?

- (a) 1
- (b) 19
- (c) $\frac{38}{3}$
- (d) $\frac{57}{2}$

51. If $f(x) = (x - 1)^2(x + 1)^4$ then $f''(0)$ is

- (a) -30
- (b) -2
- (c) 14
- (d) 30

52. If $f(x) = x^{\ln x}$, what is $f'(2)$? (Hint: use logarithmic differentiation.)

- (a) $\frac{2^{\ln 2}}{2}$
- (b) $\frac{2^{\ln 2}(\ln 2)}{2}$
- (c) $2^{\ln 2 - 1}(\ln 2)$
- (d) $2^{\ln 2}(\ln 2)$

53. [This problem was outside the scope of this course and has been omitted.]

54. The function $f(x) = \frac{x^3}{x + 1}$, $x \neq -1$ has which of the following?

- (a) One Local Maximum
- (b) One Local Minimum
- (c) One Local Maximum and One Local Minimum
- (d) Two Local Maximums

55. Suppose the demand curve for a certain product is given by $p = 28 - x$, where p is the price per unit and x is the number of units produced. If the cost of producing the product is $C(x) = x^3 - 7x^2 + 13x + 15$, at what level of production is the profit maximized?

- (a) -1
- (b) 1
- (c) 2
- (d) 5

56. What is the average value of the function $f(x) = x^{3/2}$ over the interval $1 \leq x \leq 4$?

- (a) 62
- (b) $\frac{62}{3}$
- (c) $\frac{62}{5}$
- (d) $\frac{62}{15}$
- (e) None of the above

57. Find the consumers' surplus for the demand curve $p = 50 - .06x^2$ at the sales level $x = 20$.

- (a) 240
- (b) 320
- (c) 440
- (d) 840

58. Suppose that money is deposited continuously into a savings account at an annual rate of \$1000. The account pays 6% interest compounded continuously. Which integral expresses the amount of money in the account after 5 years?

- (a) $\int_0^5 1000e^{-.06t} dt$
- (b) $\int_0^5 1000e^{.06t} dt$
- (c) $\int_0^5 1000e^{.06(5-t)} dt$
- (d) $\int_0^5 1000e^{.06(5t)} dt$

59. Suppose that money is deposited daily for 7 years into a savings account at an annual rate of \$4000 and the account pays 6% interest compounded continuously. The present value of this income is
- (a) Less than \$28,000
 - (b) Exactly \$28,000
 - (c) More than \$28,000