

Wednesday January 23, 2008  
Math 1B

1. a. Use integration by parts to prove the formula

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx.$$

- b. Evaluate  $\int (\ln x)^3 dx$ .

2. Evaluate the following integrals

(a)  $\int_0^\pi \sin^3 x \cos^8 x dx$

(b)  $\int_0^\pi \sin^4 x dx$

(c)  $\int \sin^2 x \cos^2 x dx$

(d)  $\int \tan^2 x \sin^3 x dx$

3. Evaluate the following integrals

(a)  $\int \frac{\cos x + 1}{\cos x - 1} dx$

(b)  $\int \frac{\sin x + \cos x}{\sin 2x} dx$

4. Evaluate the following integrals:

(a)  $\int \sqrt{9 - e^{2t}} dx$

(b)  $\int \frac{dx}{\sqrt{x^2 - 4x - 5}}$

(c)  $\int \frac{dx}{x+x^3}$ .

5. Let  $a$  be a positive real number and  $f(x) = \sqrt{a^2 - x^2}$ .

(a) For which values of  $x$  is  $f(x)$  defined? Sketch the domain of  $f$  on a number line.

(b) Draw a right triangle and decide which edges best represent  $x$  and  $f(x)$ . Label all three edges with an appropriate value. Express  $\sin \theta$ ,  $\tan \theta$  and  $\sec \theta$  (where  $\theta$  is an acute angle of your triangle) in terms of the values written on the edges.

(c) Write  $x$  as a function of  $\theta$ ,  $x = j(\theta)$ . What is the domain and range of the function  $j$ ?

(d) Does the function  $f(j(\theta))$  have the same domain and range as the function  $f(x)$ ?

(e) Now integrate  $\int \frac{dx}{f(x)}$  using the substitution  $x = j(\theta)$ .

6. Using integration, show that the area of a circle with radius  $r$  is  $\pi r^2$ . A good picture will help.

7. Evaluate

$$\int \frac{x^2 dx}{(x^2 + a^2)^{3/2}}$$

first by a trig substitution and then by the hyperbolic substitution  $x = a \sinh t$